

The most beautiful of theories

After publishing the theory of special relativity, Einstein becomes a renowned physicist and receives offers of work from numerous universities. But something troubles him: special relativity does not square with what was known about gravity. He realizes this while writing a review on his theory, and wonders whether the venerable theory of the 'universal gravity' of the father of physics, Newton, should not be reconsidered as well, to make it compatible with his relativity.

The origin of the problem is easy to understand. Newton had tried to explain why things fall and planets revolve. He had imagined a 'force' that draws all bodies towards one another: the 'force of gravity'. How this force managed to draw distant things together without anything between them was not understood. Newton himself, as we have seen, had suspected that in the idea of a force acting between distant bodies that do not touch there was something missing; and that in order for the Earth to attract the Moon something that could transmit this force had to be there between the two. Two hundred years later, Faraday had found the solution – not for the force of gravity, but for the electric

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and magnetic forces: the field. Electric and magnetic fields 'carry around' the electric and magnetic force.

It's clear, at this stage, to any reasonable person, that the force of gravity must have its Faraday lines as well. It's clear also, by analogy, that the force of attraction between the Sun and the Earth, or between the Earth and falling objects, must be attributed to a field – in this case, a gravitational field. The solution discovered by Faraday and Maxwell to the question as to what carries the force must reasonably be applied not only to electricity but also to gravity. There must be a gravitational field, and some equations analogous to Maxwell's, capable of describing how Faraday's gravitational lines move. In the first years of the twentieth century this is clear to any sufficiently reasonable person; that is to say, only to Albert Einstein.

Einstein, fascinated since adolescence by the electromagnetic field that pushed the rotors in his father's power stations, begins to look into this gravitational field and search for what kind of maths could describe it. He immerses himself in the problem. It would take ten years to resolve. Ten years of manic studies, attempts, mistakes, confusion, brilliant ideas, wrong ideas, a long series of articles published with incorrect equations, further mistakes and stress. Finally, in 1915, he commits to print an article containing the complete solution, which he names the General Theory of Relativity: his masterpiece. It is Lev Landau, the most outstanding theoretical physicist of the Soviet Union, who called it 'the most beautiful of theories'.

The reason for the beauty of the theory is not hard to see. Instead of simply inventing the mathematical form of the gravitational field and seeking to devise the equations for it, Einstein fishes out the other unresolved question in the

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furthest depths of Newton's theory and combines the two questions.

Newton had returned to Democritus's idea, according to which bodies move in *space*. This *space* had to be a large, empty container, a rigid box for the universe; an immense scaffolding in which objects run in straight lines, until a force causes them to curve. But what is this 'space' which contains the world made of? What *is* space?

To us, the idea of space seems natural, but it is our familiarity with Newtonian physics that makes it so. If you think about it, empty space is not part of our experience. From Aristotle to Descartes, that is to say, for two millennia, the Democritean idea of space as a peculiar entity, distinct from things, had never been seen as reasonable. For Aristotle, as for Descartes, things have extension: extension is a property of things; extension does not exist without something being extended. I can take away the water from a glass, but air will fill it. Have you ever seen a really empty glass?

If between two things there is *nothing*, Aristotle reasoned, then there is nothing. How can there be at the same time something (space) and nothing? What is this empty space within which particles move? Is it something, or is it nothing? If it is nothing, it doesn't exist, and we can do without it. If it is something, can it be true that its only property is to be there, *doing* nothing?

Since antiquity, the idea of empty space, halfway between a thing and a non-thing, had troubled thinkers. Democritus himself, who had placed empty space at the basis of his world where atoms course, certainly wasn't crystal clear on the issue: he wrote that empty space is something 'between being and non-being': 'Democritus postulated the full and the empty, calling one "Being", and the other "Non-Being";

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says Simplicius.¹ Atoms are being. Space is non-being – a 'non-being' that, nevertheless, exists. It is difficult to be more obscure than this.

Newton, who resuscitated the Democritean idea of space, had tried to patch things up by arguing that space was God's *sensorium*. No one has ever understood what Newton meant by 'God's sensorium', perhaps not even Newton himself. Certainly, Einstein, who gave little credit to the idea of a God (with or without a sensorium), except as a playful rhetorical device, found Newton's explanation of the nature of space utterly unconvincing.

Newton struggled considerably to overcome the scientists' and philosophers' resistance to his reviving the Democritean concept of space; at first nobody took him seriously. Only the extraordinary efficacy of his equations, which turned out to predict always the correct outcome, ended up silencing criticism. But doubts concerning the plausibility of the Newtonian concept of space persisted, and Einstein, who read philosophers, was well aware of them. Ernst Mach, whose influence Einstein readily acknowledged, was the philosopher who highlighted the conceptual difficulties of the Newtonian idea of space – the same Mach who did not believe in the existence of atoms. (A good example, incidentally, of how the same person can be short-sighted in one respect and far-seeing in another.)

Thus, Einstein addresses not one but two problems. First, how can we describe the gravitational field? Second, what is Newton's space?

And it's here that Einstein's extraordinary stroke of genius occurs, one of the greatest flights in the history of human thinking: what if the gravitational field turned out actually to *be* Newton's mysterious space? What if Newton's space

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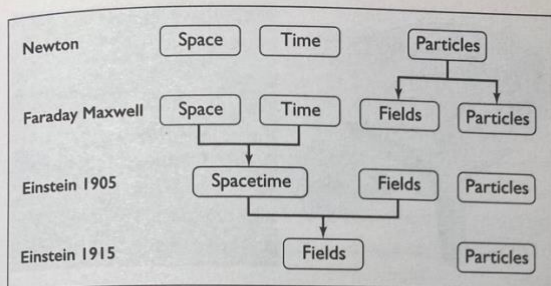


Figure 3.5 What is the world made of?

was nothing more than the gravitational field? This extremely simple, beautiful, brilliant idea is the theory of general relativity.

The world is not made up of space + particles + electromagnetic field + gravitational field. The world is made up of particles + fields, and nothing else; there is no need to add space as an extra ingredient. Newton's space *is* the gravitational field. Or vice versa, which amounts to saying the same thing: the gravitational field is space (figure 3.5).

But, unlike Newton's space, which is flat and fixed, the gravitational field, by virtue of being a field, is something which moves and undulates, subject to equations – like Maxwell's field, like Faraday's lines.

It is a momentous simplification of the world. Space is no longer different from matter. It is one of the 'material' components of the world, akin to the electromagnetic field. It is a real entity which undulates, fluctuates, bends and contorts.

We are not contained within an invisible, rigid scaffolding;

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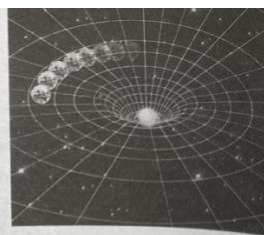


Figure 3.6 The Earth turns around the Sun because spacetime around the Sun is curved, rather like a bead which rolls on the curved wall of a funnel.

we are immersed in a gigantic, flexible mollusc (the metaphor is Einstein's). The Sun bends space around itself, and the Earth does not circle around it drawn by a mysterious distant force but runs straight in a space that inclines. It's like a bead which rolls in a funnel: there are no mysterious forces generated by the centre of the funnel, it is the curved nature of the funnel wall which guides the rotation of the bead. Planets circle around the Sun, and things fall, because space around them is curved (figure 3.6).

A little more precisely, what curves is not space but spacetime – that spacetime which, ten years previously, Einstein himself had shown to be a structured whole rather than a succession of instants.

This is the idea. Einstein's only problem was to find the equations to make it concrete. How to describe this bending of spacetime? And here Einstein is lucky: the problem had already been solved by the mathematicians.

The greatest mathematician of the nineteenth century,

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Figure 3.7 A curved (bidimensional) surface.

Carl Friedrich Gauss, the 'prince of mathematicians', had written maths to describe curved surfaces, such as the surfaces of hills, or such as the one portrayed in figure 3.7.

Then he had asked a talented student of his to generalize this maths to curved spaces in three or more dimensions. The student, Bernhard Riemann, produced a ponderous doctoral thesis of the kind that seems completely useless.

Riemann's result was that the properties of a curved space (or spacetime) in any dimension are described by a particular mathematical object, which we now call Riemann curvature and indicate with the letter 'R'. If you think of a landscape of plains, hills and mountains, the curvature *R* of the surface is zero in the plains, which are flat – 'without curvature' – and different from zero where there are valleys and hills; it is at its maximum where there are pointed peaks of mountains, that is to say, where the ground is least flat, or most curved. Using Riemann's theory, it is possible to describe the shape of curved spaces in three or four dimensions.

With a great deal of effort, seeking help from friends better versed in mathematics than himself, Einstein learns Riemann's maths – and writes an equation where *R* is proportional to the

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energy of matter. In words: spacetime curves more where there is matter. That is it. The equation is the analogue of the Maxwell equations, but for gravity rather than electricity. The equation fits into half a line, and there is nothing more. A vision – that space curves – becomes an equation.

But within this equation there is a teeming universe. And here the magical richness of the theory opens up into a phantasmagorical succession of predictions that resemble the delirious ravings of a madman but which have all turned out to be true. Even up to the beginning of the 1980s, almost nobody took the majority of these fantastical predictions entirely seriously. And yet, one after another, they have all been verified by experience. Let's consider a few of them.

To begin with, Einstein recalculates the effect of a mass like the Sun on the curvature of the space that surrounds it, and the effect of this curvature on the movements of the planets. He finds the movements of the planets as predicted by Kepler's and Newton's equations, but not exactly: in the vicinity of the Sun, the effect of the curvature of space is stronger than the effect of Newton's force. Einstein computes the movement of Mercury, the planet closest to the Sun and hence the one for which the discrepancy between the predictions of his and Newton's theories is greatest. He finds a difference: the point of the orbit of Mercury closest to the Sun moves every year 0.43 seconds of arc more than that predicted by Newton's theory. It is a small difference, but, within the scope of what astronomers were able to measure, and comparing the predictions with the observations of astronomers, the verdict is unequivocal: Mercury follows the trajectory predicted by Einstein, not the one predicted by Newton. Mercury, the fleet-footed messenger of the gods, the god of the winged sandals, follows Einstein, not Newton.

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Einstein's equation, then, describes how space curves very close to a star. Due to this curvature, light deviates. Einstein predicts that the Sun causes light to curve around it. In 1919 the measurement is achieved; a deviation of light is measured which turns out to be exactly in accordance with the prediction.

But it is not only space that curves: time does, too. Einstein predicts that time on Earth passes more quickly at higher altitude, and more slowly at lower altitude. This is measured, and also proves to be the case. Today we have extremely precise clocks, in many laboratories, and it is possible to measure this strange effect even for a difference in altitude of just a few centimetres. Place a watch on the floor and another on a table: the one on the floor registers less passing of time than the one on the table. Why? Because time is not universal and fixed, it is something which expands and shrinks, according to the vicinity of masses: the Earth, like all masses, distorts spacetime, slowing time down in its vicinity. Only slightly – but two twins who have lived respectively at sea-level and in the mountains will find that, when they meet up again, one will have aged more than the other (figure 3.8).

This effect offers an interesting explanation as to why things fall. If you look at a map of the world and the route taken by an aeroplane flying from Rome to New York, it does not seem to be straight: the aeroplane makes an arc towards the north. Why? Because, the Earth being curved, crossing northwards is shorter than keeping to the same parallel. The distances between meridians are shorter the more northerly you are; therefore, it is better to head northwards, to shorten the route (figure 3.9).

Well, believe it or not, a ball thrown upwards falls downwards for the same reason: it 'gains time' moving higher up,

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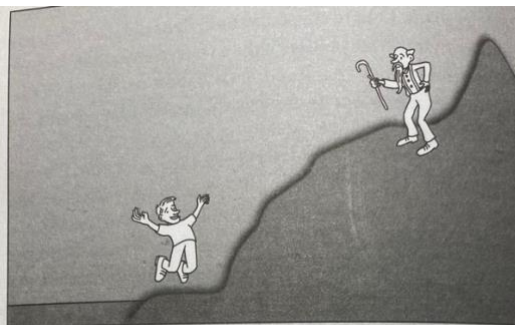


Figure 3.8 Two twins spend their time one at sea-level and the other in the mountains. When they meet up again, the twin who lived in the mountains is older. This is the gravitational dilation of time.

because time passes at a different speed up there. In both cases, aeroplane and ball follow a straight trajectory in a space (or spacetime) that is curved (figure 3.10).*

But the predictions of the theory go well beyond these minute effects. Stars burn as long as they have available hydrogen – their fuel – then die out. The remaining material is no longer supported by the pressure of the heat and collapses under its own weight. When this happens to a large enough star, the weight is so strong that matter is squashed

* Airplane and ball follow a geodesic in a curved space. In the case of the ball, the geometry is approximately given by the metric $ds^2 = (1 - 2\Phi(x))dt^2 - dx^2$, where $\Phi(x)$ is the Newtonian potential. The effect of the gravitational field is reduced to the dilation of time with altitude. (The reader familiar with the theory will notice the curious sign inversion: the physical trajectory maximizes proper time.)

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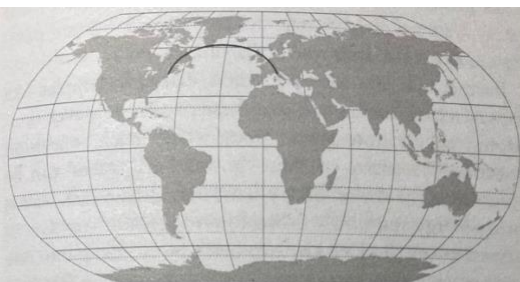


Figure 3.9 The further north you go, the smaller the distance between two meridians.

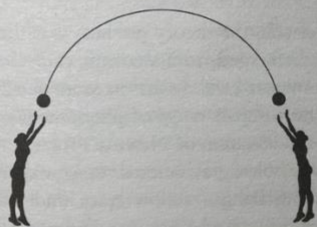


Figure 3.10 The higher up something is, the more quickly time passes for it.

down to an enormous degree and space curves so intensely as to plunge down into an actual hole. A black hole.

When I was a university student, black holes were regarded as a scarcely credible implication of an esoteric theory. Today they are observed in their hundreds and studied in detail by astronomers. One of these black holes, with a mass a million

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times greater than the Sun, is located at the centre of our galaxy – we can observe stars orbiting around it. Some, passing too close, are destroyed by its violent gravity.

Further still, the theory predicts that space ripples like the surface of the sea, and that these ripples are waves similar to the electromagnetic ones which make television possible. The effects of these 'gravitational waves' can be observed in the sky on binary stars: they radiate such waves, losing energy and slowly falling towards each other.* Gravitational waves produced by two black holes falling into one another were directly observed by an antenna on Earth in late 2015, and the announcement, given in early 2016, has once again left the world speechless. Once more, the seemingly mad predictions of Einstein's theory turn out to be precisely true.

And further still, the theory predicts that the universe is expanding and emerged from a cosmic explosion 14 billion years ago – a subject I will discuss in more detail shortly.

This rich and complex range of phenomena – bending of rays of light, modification of Newton's force, slowing down of clocks, black holes, gravitational waves, expansion of the universe, the Big Bang – follow from understanding that space is not a dull, fixed container but possesses its own dynamic, its own 'physics', just like the matter and the other fields it contains. Democritus himself would have smiled with pleasure, had he been able to see that his idea of space would turn out to have such an impressive future. It is true

* Observations of the binary system PSR B193+16 show that the two stars which revolve around one another radiate gravitational waves. These observations brought a Nobel Prize for Russell Hulse and Joseph Taylor in 1993.

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that he termed it non-being, but what he meant by being (δέν) was matter; and he wrote that his non-being, the void, nevertheless 'has a certain physics (φύσιν) and a substantiality of its own'.* How right he was.

Without the notion of fields introduced by Faraday, without the spectacular power of mathematics, without the geometry of Gauss and Riemann, this 'certain physics' would have remained incomprehensible. Empowered by new conceptual tools and by mathematics, Einstein writes the equations which describe Democritus's void and finds for its 'certain physics' a colourful and amazing world where universes explode, space collapses into bottomless holes, time slows down in the vicinity of a planet, and the boundless expanses of interstellar space ripple and sway like the surface of the sea . . .

All of this sounds like a tale told by an idiot, full of sound and fury, signifying nothing. And yet, instead, it is a glance towards reality. Or better, a glimpse of reality, a little less veiled than our blurred and banal everyday view of it. A reality which seems to be made of the same stuff our dreams are made of, but which is nevertheless more real than our clouded daily dreaming.

And all this is the result only of an elementary intuition – that spacetime and the gravitational field are one and the same thing – and a simple equation which I can't resist copying out here, even if most of my readers will certainly not be able to decipher it. I do so, anyway, in the hope that they might be able to catch a glimpse of its beautiful simplicity:

$$R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = 8\pi G T_{ab}$$

* Plutarch, *Adversus colotem*, 4, 1108. The word φύσιν means 'nature', and includes the sense 'the nature of something'.

In 1915 the equation was simpler still, because the term $+\Lambda g_{ab}$, which Einstein added two years later (and which I discuss below) did not yet exist.* R_{ab} depends on Riemann's curvature, and together with $\frac{1}{2} R g_{ab}$ represents the curvature of spacetime; T_{ab} stands for the energy of matter; G is the same constant that Newton found: the constant that determines the strength of the force of gravity.

That's it. A vision and an equation.

Mathematics or physics?

I would like to pause, before continuing with physics, to make a few observations about mathematics. Einstein was no great mathematician. He struggled with maths. He says this himself. In 1943 he replied in the following way to a nine-year-old child with the name of Barbara who wrote to him about her difficulties with the subject: 'Don't worry about experiencing difficulties with maths, I can assure you that my own problems are even more serious!'² It seems like a joke, but Einstein was not kidding. With mathematics, he needed help: he had it explained to him by patient fellow students and friends, such as Marcel Grossman. It was his intuition as a physicist that was prodigious.

During the last year in which he was completing the construction of his theory, Einstein found himself competing

* This term is called 'cosmological' because its effects occur only at an extremely large, or 'cosmological' distances. The constant Λ is called the 'cosmological constant', and its value was measured at the end of the 1990s, bringing a Nobel Prize in 2011 for the astronomers Saul Perlmutter, Brian P. Schmidt and Adam G. Riess.

Vocabulary:

1. Renowned: famous for something
2. Outstanding: clearly very much better than what is usual
3. A scaffolding: a structure of metal poles and wooden boards put against a building for workers to stand on when they want to reach the higher parts of the building
4. Utterly: completely or extremely
5. Readily: quickly, immediately, willingly, or without any problem
6. A stroke: something that happens or succeeds suddenly because of luck, intelligence, etc.
7. Momentous: very important because of effects on future events
8. A funnel: an object that has a wide round opening at the top, sloping sides, and a narrow tube at the bottom, used for pouring liquids or powders into containers with narrow necks
9. Ponderous: If a book, speech, or style of writing or speaking is ponderous, it is boring because it is too slow, long, or serious
10. Ravings: crazy statements that have no meaning
11. A discrepancy: a difference between two things that should be the same
12. A scope: the range of a subject covered by a book, programme, discussion, class, etc.:
13. Unequivocal: total, or expressed in a clear and certain way
14. A vicinity: the area around a place or where the speaker is
15. A twin: either of two children born to the same mother on the same occasion
16. A ripple: a small wave on the surface of water
17. Dull: not interesting or exciting in any way
18. To sway: to move slowly from side to side
19. A glimpse: an occasion when you see something or someone for a very short time
20. To decipher: to discover the meaning of something written badly or in a difficult or hidden way

Questions:

1. Who was the physicist before Einstein whose contribution gave rise to the concept of something existing in space due to the presence of a mass (similar to what magnets or electric charges produce)? Propose a succinct presentation of this physicist.
2. Why is the representation of a bead rolling inside a funnel helpful to understand gravity?
3. What discovery of something specific about Mercury was the confirmation of Einstein's theory of the General Theory of Relativity? When did it take place?
4. What observation involving the light of stars confirmed Einstein's theory of the General Theory of Relativity? When did it take place?
5. What observation involving black holes was another confirmation of Einstein's theory of the General Theory of Relativity? When did it take place?