Travelling-wave characteristics

This chapter introduces a new and special kind of motion: wave motion. There are two large classes of waves: mechanical and electromagnetic waves. Waves can be further classified into transverse and longitudinal waves.

Objectives

By the end of this chapter you should be able to:

- state what is meant by *wave motion*;
- distinguish between *longitudinal* and *transverse* waves;
- define *amplitude*, *wavelength*, *period* and *frequency* and state the relationship between them, $f = \frac{1}{T}$;
- state what is meant by *crest* and *trough* and identify these on a graph;
- find amplitude and period from a *displacement–time* graph;
- find amplitude and wavelength from a *displacement–position* graph;
- understand the meaning of the terms *wavefront* and *ray*;
- use $v = \lambda f$.

What is a wave?

Waves are a very special kind of motion that differs significantly from the motion we have studied in earlier chapters. To understand the difference, and to appreciate this new kind of motion, let us look at what we have learned in a somewhat different way. If a stone is thrown at a window and the window breaks, this is because the stone transferred its kinetic energy from the point at which it was thrown onto the window. The stone exerted a force on the window (transfer of momentum) and broke it. A wave is also a way of transferring energy and momentum from one place to another but *without the actual large-scale motion of a material body*. For example, light (a kind of wave) from the sun arrives on earth having travelled a large distance in a vacuum, and upon arrival warms up the earth. A soprano singing can break a crystal glass because energy and momentum have been transferred through air by a sound wave.

Light is an example of a wave that does not need a medium in which to travel. It can travel in a vacuum as well as in solids (e.g. glass) or liquids (e.g. water). Light is part of a large family of waves called *electromagnetic waves*. Sound, along with water waves, string waves, etc., belongs to a family called *mechanical waves*. These do require a medium for their propagation. Sound, for example, cannot travel in a vacuum. Sound can travel in solids and liquids as well as, of course, in gases. Similarly, water waves travel, not surprisingly, in water.

How do we describe a wave? A wave is always associated with a *disturbance* of some kind. A rope held tight is horizontal when no wave is travelling on it. By moving one end up and down, we create a disturbance and individual points on the rope are now higher or lower than their original undisturbed positions. In the case of sound, the density of air becomes successively higher or lower when a sound wave travels through the air compared with when there is no sound wave. (The case of light is a bit more complicated and we will not discuss it here.)

▶ A wave is a disturbance that travels in a medium (which can be a vacuum in the case of electromagnetic waves) transferring energy and momentum from one place to another. The direction of energy transfer is the direction of propagation of the wave.

Note that in all the examples we have talked about, there is no large-scale motion of the medium. Points on a rope oscillate up and down, and molecules in air move back and forth along the direction of a sound wave that is travelling through the air. This is local, small-scale motion; the material of the medium does not itself travel large distances.

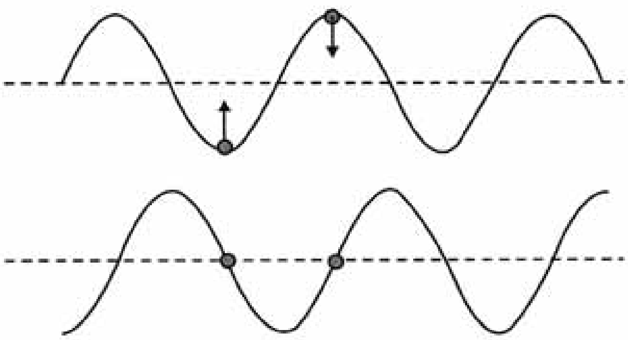

Figure 2.1 A transverse wave on a string travelling to the right. At the early time of the top picture, the parts of the string marked are at their maximum displacement above and below the equilibrium position of the string. Some time later the left part has moved up and the right part down – their motion is at right angles to the direction of motion of the wave.

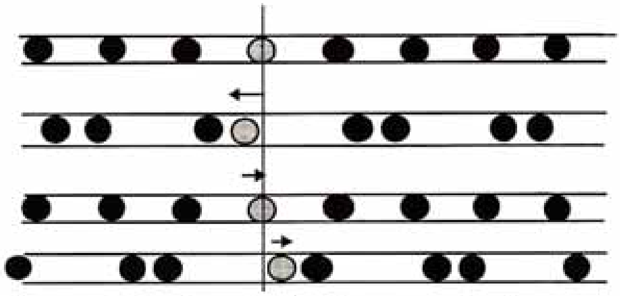

Figure 2.2 In a longitudinal wave, molecules execute simple harmonic motion along the direction of propagation of the wave.

Transverse and longitudinal waves

In addition to the division into mechanical and electromagnetic, waves can be further divided into two classes. The first class is called **transverse** and consists of those waves in which the disturbance is *at right angles* to the direction of propagation of the wave. A typical example is a wave on a string: the direction of propagation is along the string but the disturbance is at right angles to the string (see Figure 2.1). Electromagnetic waves are also transverse.

The second class is called **longitudinal** and consists of those waves in which the disturbance is *along* the direction of propagation of the wave. A typical example is sound: if we imagine that a sound wave is moving from left to right in a thin tube, the disturbance is the motion of air molecules back and forth along the tube. In Figure 2.2 the dots represent molecules of air. In the top picture the molecules are equally spaced, representing the gas in its equilibrium state. As the wave passes through the gas, the molecules move to the right or left. As they do so they create regions of higher than normal density (compressions) and regions of lower than normal density (rarefactions). For convenience we have marked one molecule grey to indicate that molecules execute simple harmonic oscillations about their equilibrium positions (dotted line for the grey molecule). Figure 2.3 shows the compressions and rarefactions that occur in the medium in which the wave moves.

Figure 2.3 The motion of the molecules causes compressions and rarefactions in the medium in which the wave moves.

Wave pulses

To help us to understand waves we will start with a simple case, that of a wave pulse. If you tie one end of a rope to a wall and move the other end sharply up and then back down to its starting position, you will produce a wave pulse that will travel along the rope. It looks like Figure 2.4 (this is idealized to a triangular pulse – the real pulse would be curved):

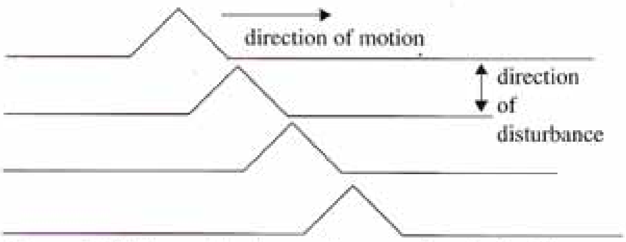

Figure 2.4 The pulse is moving to the right. The disturbance is normal to the direction of motion.

If your hand is first moved down below the rope then back up to the starting point, continues up above the rope and finally back down to the starting point, the pulse will look like Figure 2.5.

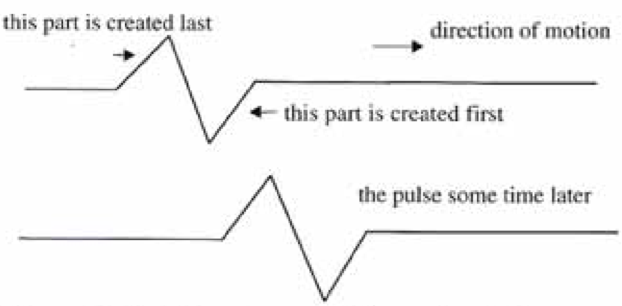

Figure 2.5 A full pulse travelling to the right.

It takes a certain time for this disturbance to move along the rope (i.e. for this wave pulse to reach another point in the rope). The wave pulse travels with a certain **speed** down the rope. In the case of the wave pulse on the rope, the speed of the pulse is determined not by the way in which the pulse was created (big or small pulse, wide or narrow) nor by how fast or slow your hand moved the rope; rather, it is determined by the tension T in the rope and the mass per unit length $\mu = \frac{m}{L}$ of the rope. Although not required for examination purposes, it is good to know the following:

> ▶ The speed of the pulse on the string is given by
>
> $$v = \sqrt{\frac{T}{\mu}}$$
>
> The speed of the wave is determined by the properties of the medium and not by how the wave is created.

The greater the tension in the medium, the greater the speed of the wave produced. You can convince yourself that the speed is greater when the tension is greater by creating pulses on a slinky, which can be kept at various tensions by having it stretched by different amounts. You will then also see that v is independent of the shape of the pulse you produce and of how fast you produced it.

The statement that the speed of the pulse is independent of the amplitude is true provided the amplitude is not too big. If the amplitude is big, then the string is more stretched and thus the tension is greater, implying a greater pulse speed. Not too big an amplitude means not big compared with the length of the string.

Travelling waves

In the previous section we saw how a single pulse can be produced on a stretched rope. We can create a *travelling wave* if we now produce one pulse after another. If, in addition, the agent forcing the rope up and down executes simple harmonic motion, then the wave will look like a sine wave (also called a harmonic wave) – see Figure 2.6 (top).

If the sequence of pulses produced are square pulses, then the wave generated is a travelling square wave – see Figure 2.6 (bottom).

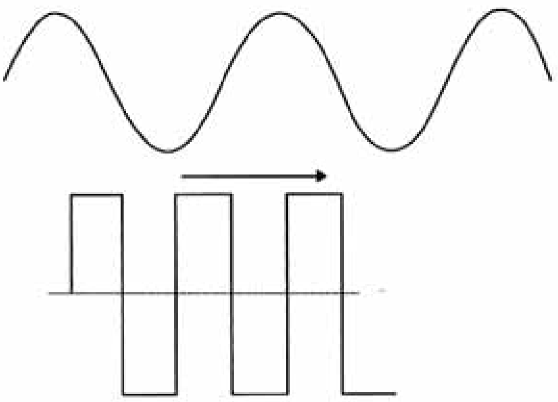

Figure 2.6 A periodic sine wave and a periodic square wave.

Harmonic waves are very important because any periodic disturbance can be expressed as a sum (superposition) of a number of harmonic waves. This is a general theorem in mathematics known as Fourier's theorem.

Harmonic waves

A simple way of producing harmonic waves is to attach one end of a rope to a tuning fork, as shown in Figure 2.7. If the tuning fork is then made to oscillate, one full wave will be produced on the rope after a time equal to the **period** of the tuning fork.

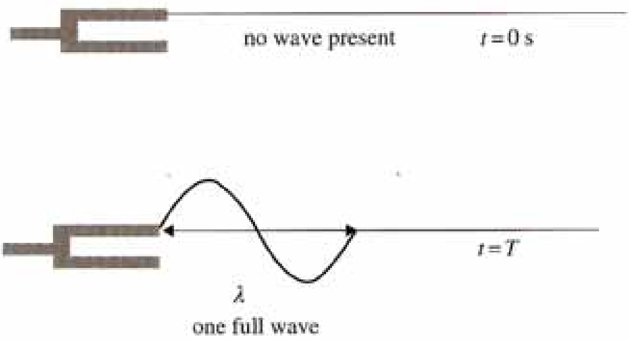

Figure 2.7 A full wave is produced in a time equal to one period.

▶ The length of a full wave is called the wavelength, λ, and the time needed to produce one full wave (or the duration of a full wave) is the period, T.

After a second full period, a second full wave will be produced (see Figure 2.8). The original full wave has moved forward a distance equal to the **wavelength**.

Figure 2.8 Two full waves are produced in sequence by the oscillating tuning fork.

▶ It thus follows that, since the wave moves forward a distance equal to a wavelength in a time equal to one period, the speed of the wave is given by

$$v = \frac{\lambda}{T}$$

Since one full wave is produced in a time of T s, it follows that the number of full waves produced in 1 s is $1/T$. This is the **frequency**.

▶ The number of full waves produced in 1 s is called the frequency of the wave, $f = \frac{1}{T}$. The unit of frequency is the inverse second, which is given the special name hertz (Hz). In terms of frequency the wave speed is thus

$$v = \lambda f$$

Waves can be represented graphically. This is a bit complicated because a wave depends on distance (where along the wave are we looking?) as well as time (at what time are we looking at the wave?). First we have to decide how we will quantify the 'disturbance' of the wave. For a wave on a string the obvious choice is to measure the height of a point on the string above or below the undisturbed position of the string. The disturbance here is thus the *displacement* of a point on the string and is measured in units of length. We normally denote this displacement by y.

which will be a function of distance (x) and time (t).

In the case of sound, the disturbance may be associated with the density of the medium through which sound propagates. We may then define the difference $y_\rho = \rho - \rho_0$ as the *displacement* of density (ρ) relative to the equilibrium density of the medium when no sound is present in it (ρ_0). The displacement here has units of density and is also a function of position and time. In the case of sound, we could equally well define displacement as the difference $y_p = p - p_0$, which is the difference between the pressure of the medium when sound is present and the equilibrium pressure when no sound is present. Displacement would then have units of pressure.

This discussion can be generalized to all waves. All waves have a displacement that is the difference of some quantity and the equilibrium value of that quantity when no wave is present. The displacement of any wave is a function of position and time. We may therefore represent waves in graphs of displacement versus position (distance) and graphs of displacement versus time.

Let us consider a wave propagating along a string from left to right. The left end of the string is represented by $x = 0$ m and any other point on the string is specified by giving its corresponding x coordinate (see Figure 2.9).

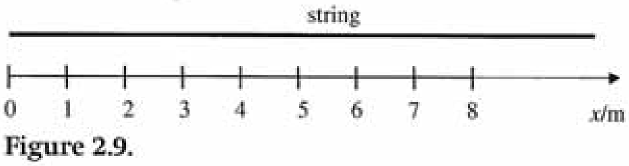

Figure 2.9.

As the wave propagates along the string, we would like to know the displacement at each point on the string *at a specific point in time*. This is given by a graph of displacement versus position – Figure 2.10.

The first important piece of information from such a graph is the wavelength.

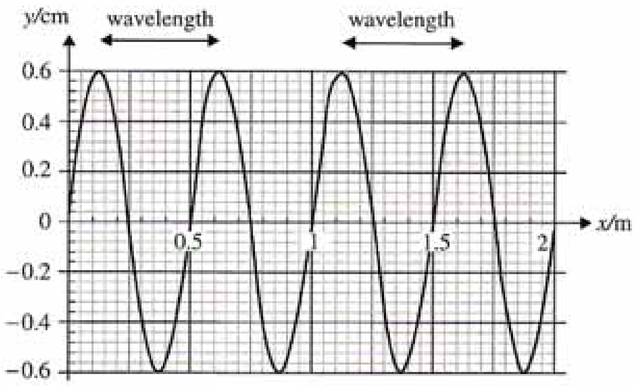

Figure 2.10 A graph of displacement versus position tells us the disturbance of any point on the string at a specific moment in time.

▶ Graphs showing the variation of displacement with position enable us to determine the wavelength of the wave as the distance from peak to next peak (or the length of a full wave).

This graph also tells us that at the point on the string that is 0.5 m from the string's left end the displacement is zero at some point in time. At that *same point in time* at a point 1.125 m from the left end the displacement is 0.6 cm, etc. Thus, a graph of displacement versus position is like a *photograph* of the string taken at a particular time. If we take a second photograph of the string some time later, the string will look different because the wave has moved in the meantime. It might look like Figure 2.11.

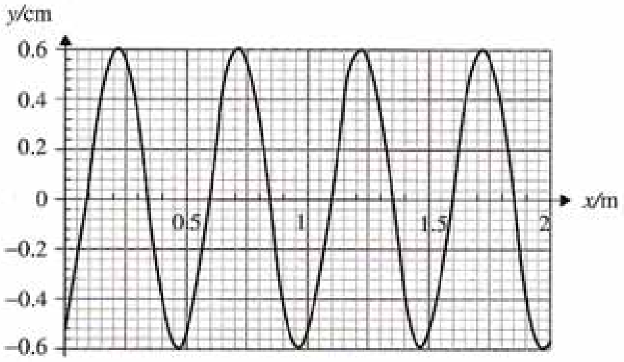

Figure 2.11 A graph of the disturbance of any point on the string at a later moment in time. Note that every point has a different disturbance from that shown in Figure 2.10.

We see that the displacement at $x = 0$ m that was zero in the first photograph is not zero now. It is about −0.5 cm. The displacement at a particular point on the string changes as time goes on and thus we can graph it as a function of time.

Figure 2.12 shows how the displacement of a particular point on the string (the point $x = 0$ m to be precise) varies as time goes on. This is a graph that shows the variation of displacement with time.

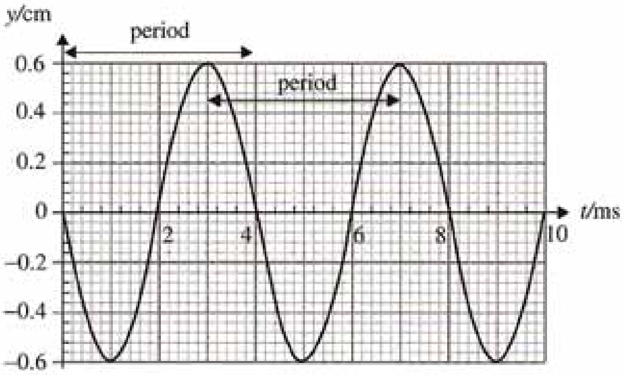

Figure 2.12 A graph of displacement versus time tells us the disturbance of a specific point on the string as time goes on.

▶ Graphs showing the variation of displacement with time enable us to determine the period of the wave as the time from peak to next peak (or the duration of a full wave).

From these graphs we can deduce the following information about the wave. From *any* graph we see that the maximum displacement of the wave is 0.6 cm. The *wavelength* of the wave can be determined by looking at a *displacement–position graph*. From Figure 2.10 it thus follows that $\lambda = 0.5$ m. To find the *period* we must look at a *displacement–time graph*. From Figure 2.12 we find $T = 4.0$ ms. Hence, the frequency is 250 Hz and the speed of the wave is 125 m s^{-1}. (Note that by comparing Figures 2.10 and 2.11 we see that the wave moved forward a distance of 0.1 m. Since the speed of the wave is 125 m s^{-1}, it follows that the photograph of Figure 2.11 was taken $0.1/125$ s $= 0.8$ ms later than that of Figure 2.10.)

Consider now the wave of Figure 2.13. We deduce that the disturbance is a pressure measured in kPa. However, in this graph the experimenter has not plotted the difference of pressure and the equilibrium value of the pressure. We may then deduce that the pressure in the medium when no wave travels through (the equilibrium pressure) is 4.0 kPa. We may also deduce that the maximum displacement (the amplitude) is 0.5 kPa. The wavelength is 4.0 m and in the absence of a displacement–time graph we can say nothing about the period or frequency, and hence speed, of this wave. On the other hand, if we are given the additional information that this is a sound wave of speed 340 m s^{-1}, then we deduce that the frequency is 85 Hz and so the period is 11.8 ms.

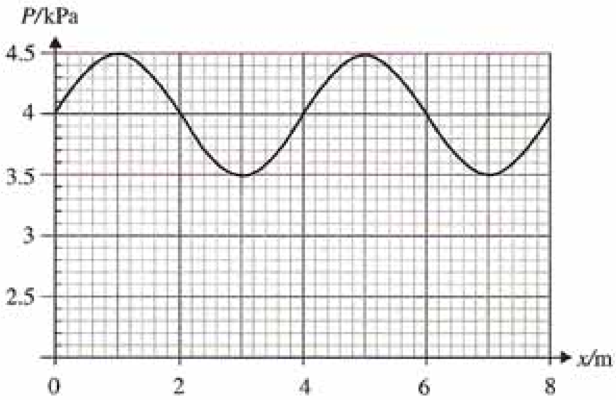

Figure 2.13 A wave in which the disturbance is about a non-zero value.

The wave of Figure 2.14 is an electromagnetic wave in which the displacement is the electric field measured in volts per metre. The amplitude is 0.2 V m^{-1} and the period is 3×10^{-15} s. The frequency is thus 3.33×10^{14} Hz. If we are told further that this wave moves in a vacuum then we know that the speed of such a wave

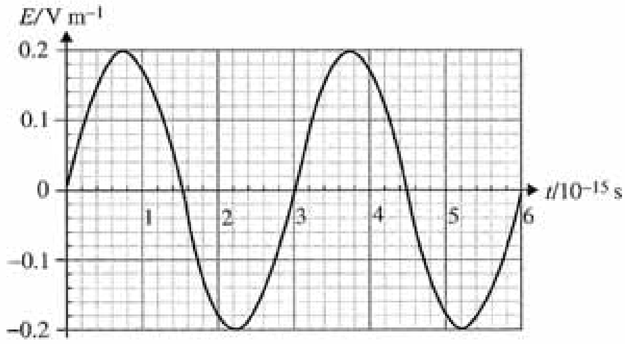

Figure 2.14 An electromagnetic wave as a function of time.

is 3×10^8 m s^{-1} and so the wavelength is 9.0×10^{-7} m.

▶ The wavelength (which we defined to be the length of one full wave) is also equal to the distance between successive crests or successive troughs in a displacement–position graph.

The period (which we defined to be the duration of one full wave) is also equal to the time between successive crests or successive troughs in a displacement–time graph.

The wavelength and frequency are two of the characteristics of a wave. A third characteristic is **amplitude**.

▶ The amplitude of a wave is defined to be the maximum displacement of the wave away from the position when no wave is present.

The amplitude of a wave is a measure of the energy the wave carries. In general, the energy carried is proportional to the square of the amplitude, which means that (all other things being equal) a water wave of amplitude 2.0 m carries four times as much energy as a water wave of amplitude 1.0 m.

In the first diagram of Figure 2.15 the amplitude of the wave is 2.0 cm. In the second it is 2.0 cm as well. The dotted line at 4.0 cm shows the equilibrium position, when no wave is present. The 4.0 cm might represent the height of a bit of water in a container. When no waves are present on the surface of the water, all points on the surface are 4.0 cm from the bottom of the container. When a small water wave is established in the container, the distance of various points on the surface varies as shown in the diagram. As the amplitude is the maximum displacement away from the equilibrium position, it is thus 2.0 cm.

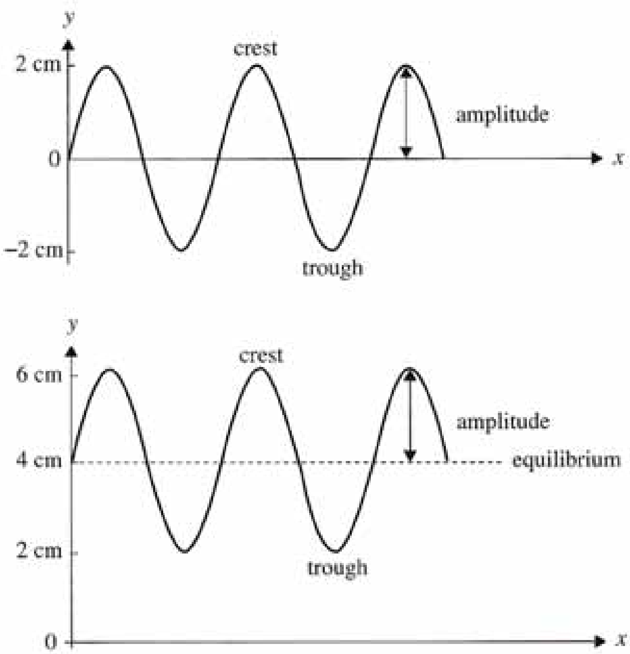

Figure 2.15 Diagrams showing the amplitude, crests and troughs of a travelling wave. In the second case, the equilibrium value is not at zero.

▶ Points on the wave with maximum displacement are called crests while those at minimum displacement are called troughs.

The diagrams in Figure 2.16 show the variation of displacement with position at various times. We see the meaning of the term travelling wave. The crests of the wave move forward.

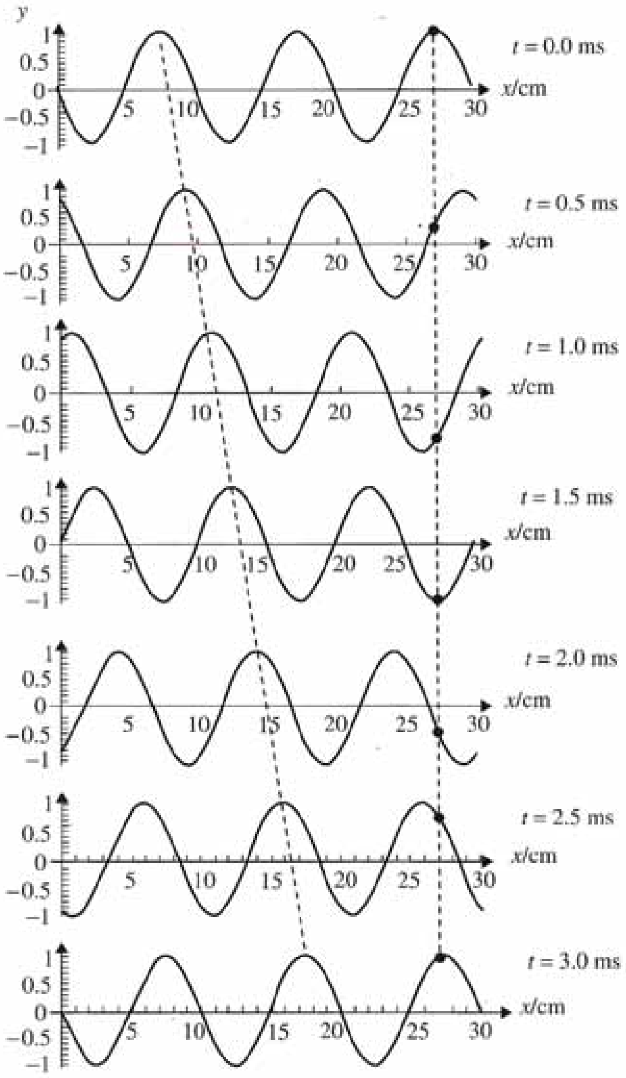

Figure 2.16 A sequence of pictures taken every 0.5 ms showing a travelling harmonic wave. Note how the peaks move forward. We have marked a point on the string to show that in a transverse wave points on the string move perpendicularly to the direction of the wave. After a full period ($T = 3.0$ ms) a picture of the rope looks like it did at the beginning ($t = 0$ ms), which is what allowed us to determine the period of the wave in the first place. The speed of the wave is 33.3 m s^{-1} (found by dividing the wavelength by the period) and the frequency is 333 Hz.

Example questions

Q1

A radio station emits at a frequency of 90.8 MHz. What is the wavelength of the waves emitted?

Answer

The waves emitted are electromagnetic waves and move at the speed of light (3×10^8 m s^{-1}). Therefore, from $v = \lambda f$ we find $\lambda = 3.3$ m.

Q2

A sound wave of frequency 450 Hz is emitted from A and travels towards B, a distance of 150 m away. Take the speed of sound to be 341 m s^{-1}. How many wavelengths fit in the distance from A to B?

Answer

The wavelength is

$$\lambda = \frac{341}{450}\,\text{m} = 0.758\,\text{m}$$

Thus the number of wavelengths that fit in the distance 150 m is

$$N = \frac{150}{0.758} = 198 \text{ wavelengths (approximately)}$$

Q3

The noise of thunder is heard 3 s after the flash of lightning. How far away is the place where lightning struck? (Take the speed of sound to be 340 m s^{-1}.)

Answer

Light travels so fast that we can assume that lightning struck exactly when we see the flash of light. If thunder is heard 3 s later, it means that it took 3 s for sound to cover the unknown distance, d. Thus

$$d = vt = 340 \times 3\,\text{m} = 1020\,\text{m}$$

Q4

Water wave crests in a lake are 5.0 m apart and pass by an anchored boat every 2.0 s. What is the speed of the water waves?

Answer

$$v = \frac{5.0}{2.0}\,\mathrm{m\,s^{-1}}$$
$$= 2.5\,\mathrm{m\,s^{-1}}$$

Q5

A toothed wheel has 300 teeth on its circumference. It rotates at 30 rpm (revolutions per minute). A piece of cardboard is placed such that it is hit by the teeth of the wheel as the wheel rotates. What is the frequency of the sound produced?

Answer

In 1 min the cardboard will be hit by a tooth $30 \times 300 = 9000$ times, which is 150 times in 1 s. The frequency of the sound is thus 150 Hz.

Q6

A railing consists of thin vertical rods a distance of 2 cm apart. A boy runs past the railing at a speed of 3 m s^{-1} dragging a stick against the rods. What is the frequency of the sound produced?

Answer

In 1 s the boy moves a distance of 3 m, or past 300/2 = 150 rods. The frequency of the sound is thus 150 Hz.

Wavefronts

Imagine a wave propagating in some direction, for example, water waves approaching the shore (see Figure 2.17).

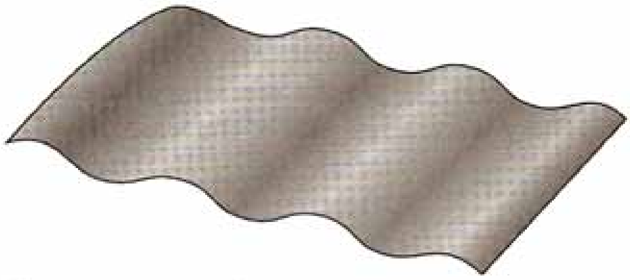

Figure 2.17 A two-dimensional wave.

The direction of the waves is horizontal, so if we imagine vertical planes going through the crests, the planes will be normal to the direction of the wave. These planes are called **wavefronts**; and lines at right angles to them are called **rays**.

> ▶ A *wavefront* is a surface through crests and normal to the direction of propagation of the wave. Lines in the direction of propagation of the wave (and hence normal to the wavefronts) are called *rays*. (See Figure 2.18.)

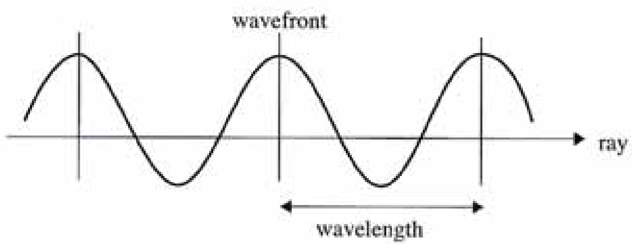

Figure 2.18 Surfaces through crests and normal to the direction of energy propagation of the wave are called wavefronts or wavecrests. Rays are mathematical lines perpendicular to the wavefronts in the direction of propagation of the wave.

(A wavefront is properly defined through the concept of *phase*. All points on a wavefront have the same phase. This will be discussed in Option G3.)

On the other hand, if we consider the surfaces going through crests of water waves caused by a stone dropped in the water, we would find that in this case the wavefronts are cylindrical surfaces (see Figure 2.19).

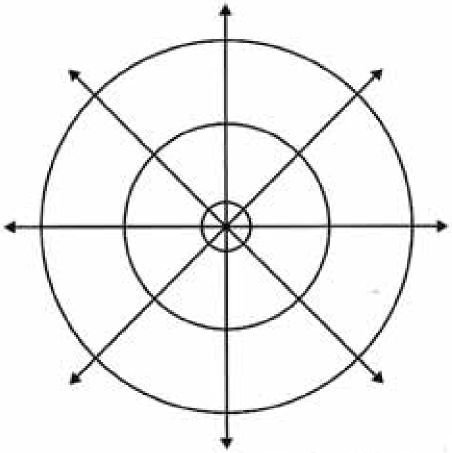

Figure 2.19 Example of cylindrical wavefronts. The cylinders go through the crests and are normal to the plane of the paper. The rays are radial lines.

Example question

Q7

A stone dropped in still water creates circular ripples that move away from the point of impact. The initial height of the ripple is about 2.4 cm and the wavelength is 0.5 m. Draw a sketch of the displacement of the ripples as a function of the distance from the point of impact.

Answer

The energy carried by the wave is distributed along the (circular) wavefronts. As the wave moves away from the point of impact, the length of the wavefront increases and so the energy per unit wavefront length decreases. Thus, the amplitude has to decrease as well. So we get the graph shown in Figure 2.20.

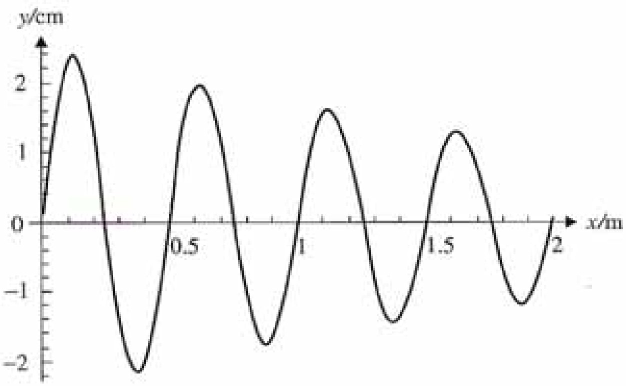

Figure 2.20.

The wavefronts of light waves leaving a point source (a very small lamp) would be spherical. We can thus speak of plane, cylindrical and spherical waves, according to the shape of the wavefronts. Note that cylindrical and spherical waves tend to become plane waves very far away from their source.

Questions

1 In football stadiums fans often create a 'wave' by standing up and sitting down again. What determines the speed of the 'wave'?

2 A number of dominoes are stood next to each other along a straight line. A small push is given to the first domino and one by one the dominoes fall over. How is this an example of wave motion? How can the speed of the wave pulse be increased? Design an experiment in which this problem can be investigated.

3 What is the wavelength that corresponds to a sound frequency of:
(a) 256 Hz;
(b) 25 kHz?
Take the speed of sound to be 330 m s^{-1}.

4 By making suitably labelled diagrams explain the terms:
(a) wavelength;
(b) period;
(c) amplitude;
(d) crest;
(e) trough.

5 The tension in a steel wire of length 0.800 m and mass 150.0 g is 120.0 N. What is the speed of transverse waves on this string? (Use $v = \sqrt{\frac{T}{\mu}}$.)

6 A string has a length of 20.0 m and is kept at a tension of 50.0 N. Its mass is 400.0 g. A transverse wave of frequency 15.0 Hz travels on this string.
(a) What is its wavelength?
(b) If the same wave is created on the same kind of string (same mass per unit length and same tension) but of double the length, what will the wavelength of the wave be? (Use $v = \sqrt{\frac{T}{\mu}}$.)

7 A stone is dropped on a still pond at $t = 0$. The wave reaches a leaf floating on the pond a distance of 3.00 m away. The leaf then begins to oscillate according to the graph shown in Figure 2.21.
(a) Find the speed of the water waves.
(b) Find the period and frequency of the wave.
(c) Find the wavelength and amplitude of the wave.

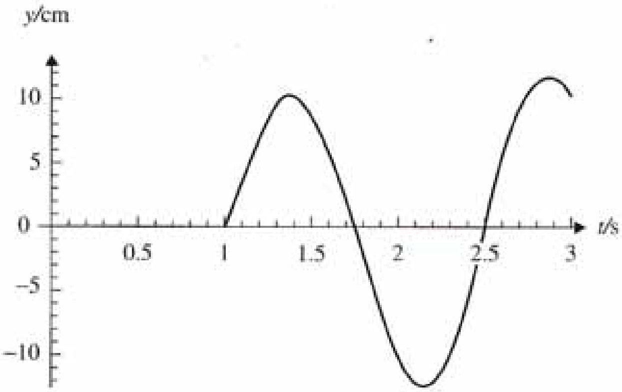

Figure 2.21 For question 7.

8 A sound wave of frequency 500 Hz travels from air into water. The speed of sound in air is 330 m s^{-1} and in water 1490 m s^{-1}. What is the wavelength of the wave in:
(a) air;
(b) water?

9 The speed of ocean waves approaching the shore is given by the formula $v = \sqrt{gh}$, where h is the depth of the water. It is assumed here that the wavelength of the waves is much larger than the depth (otherwise a different expression gives the wave speed). What is the speed of water waves near the shore where the depth is 1.0 m? Assuming that the depth of the water decreases uniformly, make a graph of the water wave speed as a function of depth from a depth of 1.0 m to a depth of 0.30 m.

10 (a) Explain, in the context of wave motion, what you understand by the term *displacement*.
(b) Using your answer in (a), explain the difference between longitudinal and transverse waves.
(c) A rock thrown onto the still surface of a pond creates circular ripples moving away from the point of impact. Why is more than one ripple created?
(d) Why does the amplitude decrease as the ripple moves away from the centre?

11 A ship sends a sonar pulse of frequency 30 kHz and duration 1.0 ms towards a submarine and receives a reflection of the pulse 3.2 s later. The speed of sound in water is 1500 m s^{-1}. Find the distance of the submarine from the ship, the wavelength of the pulse and the number of full waves emitted in the pulse.

12 Figure 2.22 shows three points on a string on which a transverse wave propagates to the right. Indicate how these three points will move in the next instant of time.

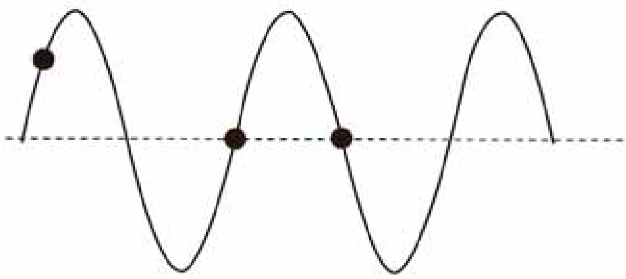

Figure 2.22 For question 12.

13 How would your answers change if the wave in question 12 were moving to the left?

14 Figure 2.23 shows a piece of cork floating on the surface of water when a wave travels through the water. On the same diagram draw the position of the cork half a wave period later.

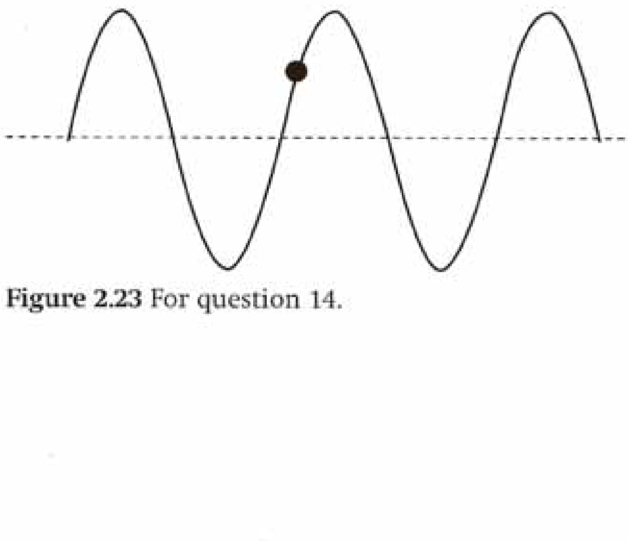

Figure 2.23 For question 14.

15 Figure 2.24 shows the same wave at two different times. The wave travels to the right and the bottom diagram represents the wave 0.2 s after the time illustrated in the top diagram. For this wave determine:

(a) the amplitude;
(b) the wavelength;
(c) the speed;
(d) the frequency.
(e) Can the graph be used to determine whether the wave is transverse or longitudinal?

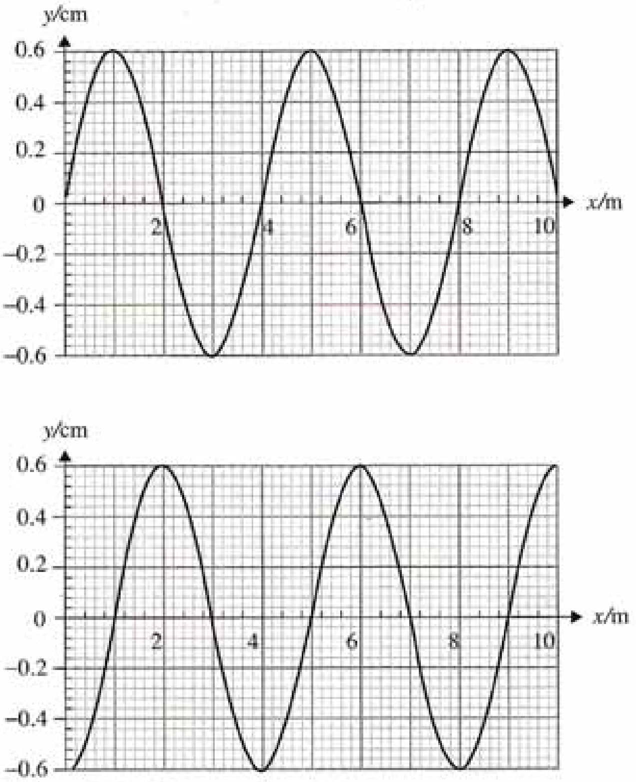

Figure 2.24 For question 15.

16 Figure 2.25 is a picture of a longitudinal wave travelling towards the right taken at a specific time. The density of the lines is proportional to the density in the medium the wave travels through.

(a) Draw this wave a very small interval of time later.
(b) Indicate on the diagram the wavelength of this wave.

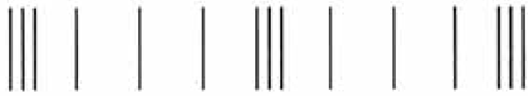

Figure 2.25 For question 16.

17 Indicate on Figure 2.26 a compression, a rarefaction and the wavelength. Draw the picture of this wave half a period later.

Figure 2.26 For question 17.

18 By drawing suitable diagrams, explain the difference between transverse and longitudinal waves.

19 In the context of wave motion explain, with the aid of a diagram, the terms:

(a) wavefront;
(b) ray.

20 An earthquake creates waves that travel in the earth's crust; these can be detected by seismic stations. Explain why three seismic stations must be used to determine the position of the earthquake. Describe *two* differences in the signals recorded by three seismic stations, assuming they are at different distances from the centre of the earthquake.