Alternating current

This chapter discusses the production of alternating current by the AC generator and the properties of alternating current. We prove the transformer equation and discuss the use of transformers in power transmission.

Objectives

By the end of this chapter you should be able to:

- appreciate that the induced emf in a uniformly rotating coil is sinusoidal;
- explain the operation and importance of the *AC generator*;
- understand the operation of the *transformer*;
- apply the *transformer equation*, $\frac{V_p}{V_s} = \frac{N_p}{N_s}$, and explain the use of transformers in power transmission;
- understand the terms *rms* and *peak current* ($I_{rms} = \frac{I_0}{\sqrt{2}}$) and *voltage* ($\mathcal{E}_{rms} = \frac{\mathcal{E}_0}{\sqrt{2}}$) and calculate the average power in simple AC circuits ($\langle P \rangle = \frac{\mathcal{E}_0 I_0}{2} = \mathcal{E}_{rms} I_{rms}$).

The AC generator

One very important application of electromagnetic induction is the AC (alternating current) generator – the method used universally to produce electricity (see Figure 8.1). The generator is in some sense a motor in reverse. A coil is made to rotate in a region of magnetic field. This can be accomplished in a variety of ways: by a diesel engine burning oil, by falling water in a hydroelectric power station, by wind power, etc.

The flux in the coil changes as the coil rotates and so an emf is produced in it. We assume that the coil has a single turn of wire around it, the magnetic field is $B = 0.4$ T, the coil has an area of 0.318 m^2 and a rotation rate of 50 revolutions per second. Then the flux in the coil changes as time goes on according to a cosine function as shown in Figure 8.2. (Time zero is taken to correspond to the coil in the

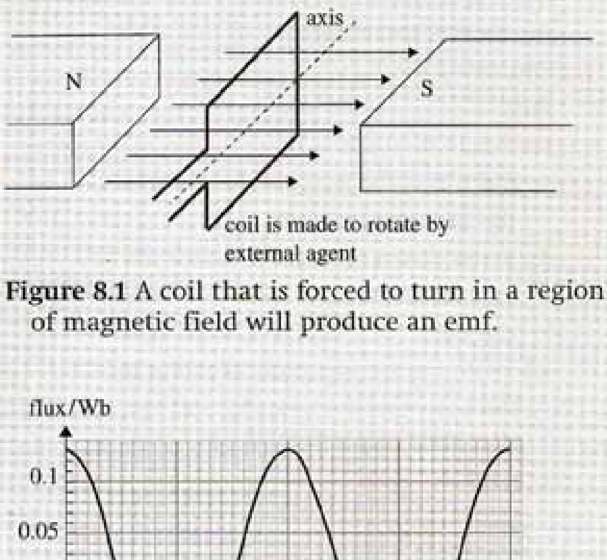

Figure 8.1 A coil that is forced to turn in a region of magnetic field will produce an emf.

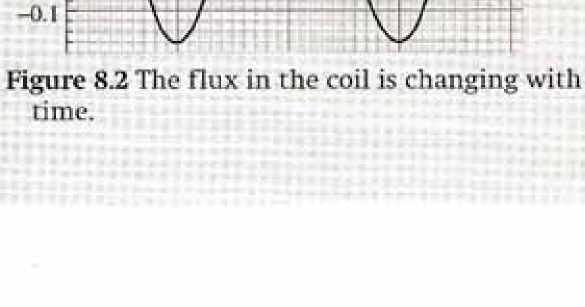

Figure 8.2 The flux in the coil is changing with time.

position of Figure 8.1 so that the flux is a maximum.)

The equation of the flux (linkage) is, in general,

$$\Phi = NBA\cos\theta$$

where θ is the angle between the magnetic field and the normal to the coil and N is the number of turns in the coil. Assuming that the coil rotates at a constant angular velocity ω, it follows that $\theta = \omega t$ and so the flux becomes

$$\Phi = NBA\cos(\omega t)$$

► By Faraday's law, the emf induced in the coil is (minus) the rate of change of the flux linkage and thus is given by

$$\mathcal{E} = -\frac{d\Phi}{dt}$$
$$\Rightarrow \mathcal{E} = \omega NBA\sin(\omega t)$$

The quantity $\mathcal{E}_0 = \omega NBA$ is the peak voltage produced by the generator. For the same numerical values as in the previous example, the emf induced is given by the graph shown in Figure 8.3 ($\omega = 2\pi f = 314.6\ s^{-1}$).

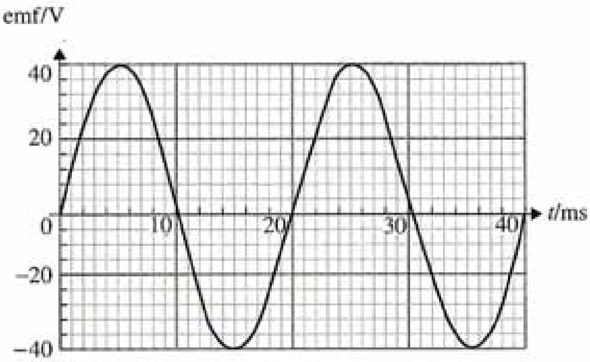

Figure 8.3 The emf induced in the loop as a function of time. The peak voltage is 40 V.

Note that the emf induced is zero whenever the flux assumes its maximum or minimum values and, conversely, it is a maximum or minimum whenever the flux is zero. The noteworthy thing here is that the voltage can be negative as well as positive. This is what is called *alternating voltage* and the current that flows in the coil is alternating current (AC). This means that, unlike the ordinary direct current (DC) that flows in a circuit connected to a battery, the electrons do not drift in the same direction but oscillate back and forth with the same frequency as that of the voltage.

The current that will flow in a circuit of resistance R can be found from

$$\begin{aligned} I &= \frac{\mathcal{E}}{R} \\ &= \frac{\mathcal{E}_0\sin(\omega t)}{R} \\ &= I_0\sin(\omega t) \end{aligned}$$

where $I_0 = \frac{\mathcal{E}_0}{R}$ is the peak current. For the emf of Figure 8.3 and a resistance of 16 Ω the current is shown in Figure 8.4.

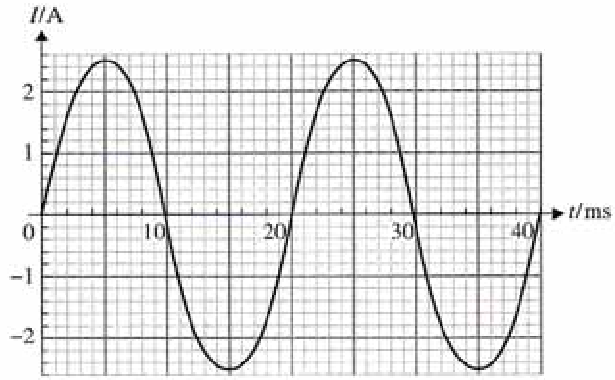

Figure 8.4 The induced current in the rotating loop. Note that the current is in phase with the emf. The peak current is found from peak voltage divided by resistance, i.e. $40/16 = 2.5$ A.

Power in AC circuits

The power generated in an AC circuit is

$$\begin{aligned} P &= \mathcal{E}I \\ &= \mathcal{E}_0 I_0\sin^2(\omega t) \end{aligned}$$

and, just like the current and the voltage, is not constant in time. It has a peak value given by the product of the peak voltage and peak current (i.e. $40 \times 2.5 = 100$ W, for the previous example). The power as a function of time is shown in Figure 8.5.

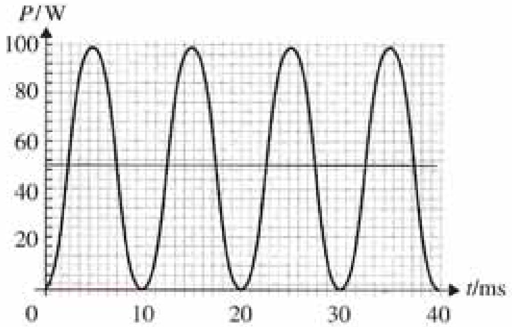

Figure 8.5 The power dissipated in a resistor as a function of time. Note that the period of one rotation of the coil is 20 ms. The power becomes zero every half rotation of the coil. The peak power is 100 W. The horizontal line indicates the average power of 50 W.

▶ It is instructive to write the expression for power in terms of the parameters of the rotating coil:

$$P = \mathcal{E}I$$
$$= \omega NBA \sin(\omega t) \times \frac{\omega NBA \sin(\omega t)}{R}$$
$$= \frac{(\omega NBA)^2}{R} \sin^2(\omega t)$$

This shows, for example, that if the speed of rotation of the coil is doubled, the power is increased by a factor of 4.

Root mean square (rms) quantities

It would be convenient to define an average voltage, average current and average power. For power this is not difficult, as power is always positive. Trying to find the average of the current or voltage, though, would give zero. In any one cycle, the voltage and current are as much positive as they are negative and so average to zero. To get around this problem we use the following trick. First, we square the current, getting a quantity that is always positive during the entire cycle. Then we find the average of this positive quantity. Finally, we take its square root. The result is called the rms value of the current (from root mean square).

How do we evaluate an rms quantity? Squaring the current gives

$$I^2 = I_0^2 \sin^2(\omega t)$$
$$= \frac{I_0^2}{2}[1 - \cos(2\omega t)]$$

where in the last step we used the identity

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}.$$

Over one cycle, the cosine term averages to zero and so the average of the square of the current is

$$\langle I^2 \rangle = \frac{I_0^2}{2}$$

(angular brackets denote an average). Thus

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

Doing exactly the same thing for the voltage results in an rms voltage of

$$\mathcal{E}_{rms} = \frac{\mathcal{E}_0}{\sqrt{2}}$$

Since

$$P = \mathcal{E}_0 I_0 \sin^2(\omega t)$$
$$= \frac{\mathcal{E}_0 I_0}{2}[1 - \cos(2\omega t)]$$

we get the following:

▶ The *average* power is

$$\langle P \rangle = \left\langle \frac{\mathcal{E}_0 I_0}{2}[1 - \cos(2\omega t)] \right\rangle$$
$$= \frac{\mathcal{E}_0 I_0}{2}$$
$$= \frac{\mathcal{E}_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}}$$
$$= \mathcal{E}_{rms} I_{rms}$$

(again the term with the cosine averages to zero over one period).

On a very non-rigorous level, we might say that dealing with rms quantities turns AC circuits into DC circuits. The product of the rms current times the rms voltage gives the *average* power in the circuit.

▶ We may also use the alternative formulae for average power

$$\langle P \rangle = R I_{\text{rms}}^2$$
$$= \frac{\mathcal{E}_{\text{rms}}^2}{R}$$

Example question

Q1

Find the rms quantities corresponding to the current and voltage of Figures 8.3 and 8.4.

Answer

The peak voltage is 40 V giving

$$\mathcal{E}_{\text{rms}} = \frac{40}{\sqrt{2}}$$
$$\approx 28 \text{ V}$$

Similarly, the peak current is 2.5 A, giving

$$I_{\text{rms}} = \frac{2.5}{\sqrt{2}}$$
$$\approx 1.8 \text{ A}$$

From Figure 8.5, the peak power is 100 W and the average power is 50 W. The product of the rms current times the rms voltage is indeed

$1.8 \times 28 = 50 \text{ W}$

The slip-ring commutator

The current must now be fed from the rotating coil into an external circuit where it can be put to use. The rotating coil is connected to the outside circuit, to which it provides current, through *slip rings*, as shown in Figure 8.6. Each of the wires leading into the coil is firmly connected to its own ring S. As the coil rotates, the ring rotates along with it, but each ring is always in contact with the same brush (B) that connects to the outside circuit. This means that since AC current is produced in the coil, AC current will be fed to the external circuit as well.

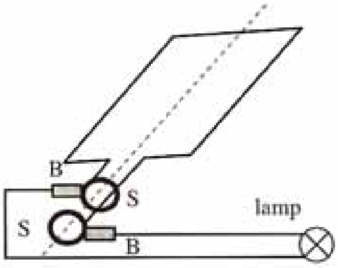

Figure 8.6 The slip-ring connection of the rotating coil to the outside circuit.

(What current would flow in an external circuit if the generator were connected to it via a *split* ring?)

The great advantage of AC voltage and current is that they allow the use of the transformer (see below).

Back-emf in the DC motor

In the DC motor, current fed into a loop that is in a magnetic field makes the coil turn as a result of the forces that develop on the sides of the loop. Because of Faraday's law an emf (the back-emf) will be induced in the loop as it begins to rotate, since there is a changing magnetic flux in the loop. By Lenz's law, this emf will oppose the change in the flux that created it. This means that a current will flow in the loop that is opposite to the current that the external battery feeds into the loop. The current in the loop will thus be less when the coil is rotating than initially, when the rotation had not yet started. The back-emf is the reason that lights sometimes dim when the motor of the refrigerator turns on. Initially the current drawn by the motor is large and only after the coil of the motor achieves a constant speed of rotation does the current drop to lower values.

The transformer

Consider two coils placed near each other as shown in Figure 8.7. The turns of both coils are wrapped around an iron core.

The first coil (the primary) has N_p turns of wire and the second (the secondary) N_s turns. If the

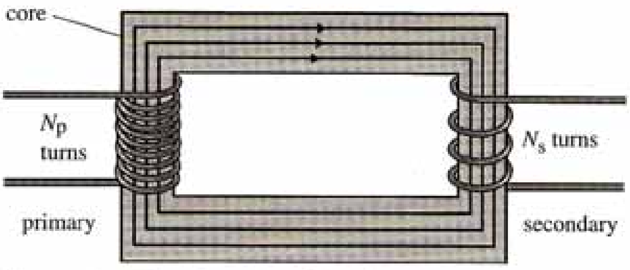

Figure 8.7 The changing flux in the secondary coil produces an emf in that coil.

primary coil is connected to an AC source of voltage, an alternating current will pass through this coil. The magnetic field this current will create will be changing in both magnitude and direction, since the current is. As the two coils are near each other, the magnetic field of the first coil enters the second coil and thus there is magnetic flux in this second coil. The purpose of the iron core is to ensure that as much of the flux produced in the primary coil as possible enters the secondary coil. Iron has the property that it confines magnetic flux and so no magnetic field lines spread out into the region outside the core. Since the magnetic field is changing, the flux is also changing and thus, by Faraday's law, there will be an *induced* emf in the secondary coil.

If the flux is changing at a rate $\frac{\Delta\Phi}{\Delta t}$ through one turn of wire, then the rate of change of flux linkage in the second coil is $N_s\frac{\Delta\Phi}{\Delta t}$, and that, therefore, is the emf induced in the secondary coil, $V_s = N_s\frac{\Delta\Phi}{\Delta t}$. Similarly, the emf, V_p, in the first coil is $V_p = N_p\frac{\Delta\Phi}{\Delta t}$. Hence

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

The arrangement just described is called a *transformer*. What we have achieved is to make a device that takes in AC voltage (V_p) in the primary coil and delivers in the secondary coil a *different* AC voltage (V_s). If the secondary coil has more turns than the primary, the secondary voltage is bigger than the primary voltage (if the secondary coil has fewer turns, the secondary voltage is smaller). *Note that the transformer works only when the voltage in the primary coil is changing.* Direct (i.e. constant) voltage fed into the primary coil would result in zero voltage in the secondary (except for the short interval of time it takes the current in the primary coil to reach its final steady value). In the case of standard AC voltage, there is a sine dependence on time with a frequency of 50 or 60 Hz. The frequency of the voltage in the secondary coil stays the same – the transformer cannot change the frequency of the voltage.

If the primary coil has a current I_p in it, then the power dissipated in the primary coil is V_pI_p. Assuming no power losses, the power dissipated in the secondary coil is the same as that in the primary and thus

$$V_pI_p = V_sI_s$$

Therefore, using $\frac{V_p}{V_s} = \frac{N_p}{N_s}$ the relationship between the currents is

$$\frac{I_p}{I_s} = \frac{N_s}{N_p}$$

(Power losses are reduced by having a laminated core rather than a single block for the core – this reduces power losses by eliminating eddy currents. Eddy currents are created in the core because the free electrons of the core move in the presence of a magnetic field. Thus these electrons are deflected into circular paths and they create small currents in the core. See Figure 8.8.)

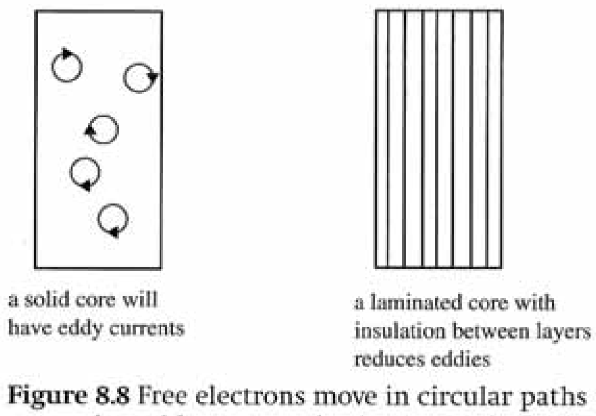

Figure 8.8 Free electrons move in circular paths creating eddy currents in the magnetic field that is established in the core. Nearly all of these currents are eliminated if the core is laminated.

Transformers and power transmission

Transformers are used in the transport of electricity from power stations, where electricity is produced, to the consumer. At any given time, a city will have a power demand, P, which is quite large (many megawatts for a large city). If the power station sends out electricity at a voltage V and a current I flows in the cables from the power station to the city and back, then

$$P = VI$$

The cables have resistance, however, and thus there is power loss $P_{loss} = RI^2$ where R stands for the total resistance of the cables. To minimize this loss it is necessary to minimize the current (there is not much that can be done about minimizing R). However, small I (I is still a few thousand amperes) means large V (recall, $P = VI$), which is why power companies supply electricity at large voltages. Transformers are then used to reduce the high voltage down to that required for normal household appliances (220 V or 110 V). (See Figure 8.9.)

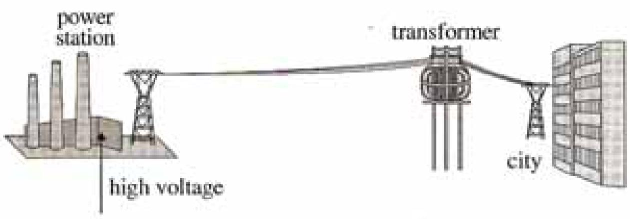

Figure 8.9 The voltage produced in the power station is high in order to reduce losses during transmission. Transformers are used to step down the voltage to what the consumer requires.

Example question

Q2

A power company produces 500 MW of electricity at a voltage of 1.2×10^5 V. The total resistance of the cables leading to and from a town is 4 Ω. How much current flows from the power station? What is the percentage loss of power in the cables? If the electricity were transmitted at the lower voltage of 0.8×10^5 V, what would the power loss be?

Answer

From $P = VI$ the current is

$$I = \frac{500 \times 10^6}{1.2 \times 10^5} = 4.2 \times 10^3 \text{A}$$

The power loss in the cables is

$$P_{loss} = RI^2 = 4.0 \times (4.2 \times 10^3)^2 = 7.1 \times 10^7 \text{ W} = 71 \text{ MW}$$

This corresponds to a power loss of $71/500 \times 100\% = 14\%$ of the produced power. With the lower voltage the current is

$$I = \frac{500 \times 10^6}{0.8 \times 10^5} = 6.2 \times 10^3 \text{A}$$

The power lost is then

$$P = RI^2 = 4.0 \times (6.2 \times 10^3)^2 = 1.5 \times 10^8 \text{ W} = 150 \text{ MW}$$

The percentage of power lost is now $150/500 \times 100\% = 30\%$.

Questions

1 A transformer has 500 turns in its primary coil and 200 in the secondary coil.
   (a) If an AC voltage of 220 V and frequency 50 Hz is established in the primary coil, find the voltage and frequency induced in the secondary coil.
   (b) If the primary current is 6.0 A, find the current in the secondary coil assuming an efficiency of 70%.

2 A 300 MW power station produces electricity at 80 kV, which is then supplied to consumers along cables of total resistance 5.0 Ω.
   (a) What percentage of the produced power is lost in the cables?
   (b) What does the percentage become if the electricity is produced at 100 kV?

3 The rms voltage output of a generator is 220 V. The coil is a square of side 20.0 cm, has 300 turns of wire and rotates at 50 revolutions per second. What is the magnetic field?

4 Figure 8.10 shows the variation, with time, of the magnetic flux linkage through a loop. What is the rms value of the emf produced in the loop?

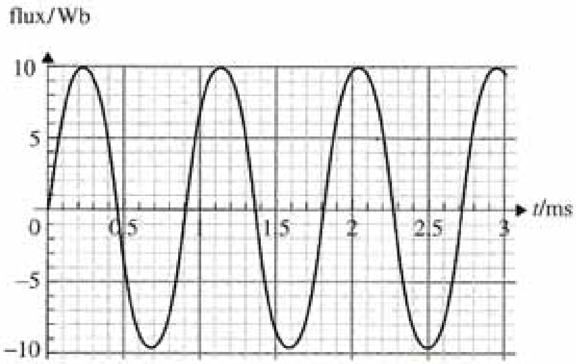

Figure 8.10 For question 4.

5 A power station produces 150 kW of power, which is transmitted along cables of total resistance 2.0 Ω. What fraction of the power is lost if it is transmitted at:

(a) 1000 V;

(b) 5000 V?

6 If the connection of a rotating generator coil to the outside circuit were made through a split ring (as discussed in the case of the DC motor), what sort of current would flow in the external circuit?

7 Figure 8.11 shows the variation with time of the power dissipated in a resistor when an alternating voltage from a generator is established at its ends. Assume that the resistance is constant at 2.5 Ω.

(a) Find the rms value of the current.

(b) Find the rms value of the voltage.

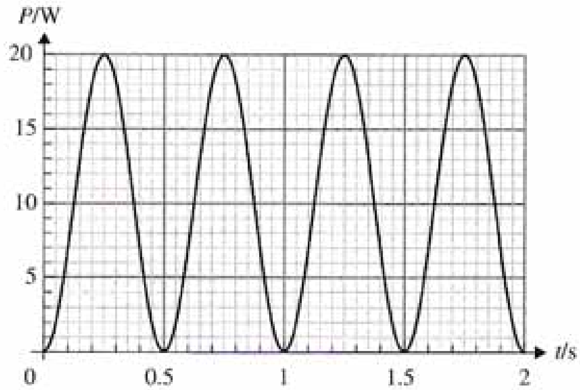

Figure 8.11 For question 7.

(c) Find the period of rotation of the coil.

(d) The coil is now rotated at double the speed. Draw a graph to show the variation with time of the power dissipated in the resistor.

8 Figure 8.12 shows the variation of the flux in a coil as it rotates in a magnetic field with the angle between the magnetic field and the normal to the coil.

(a) Draw a graph to show the variation of the induced emf with angle.

The same coil is now rotated at double the speed in the same magnetic field. Draw graphs to show:

(b) the variation of the flux with angle;

(c) the variation of the induced emf with angle.

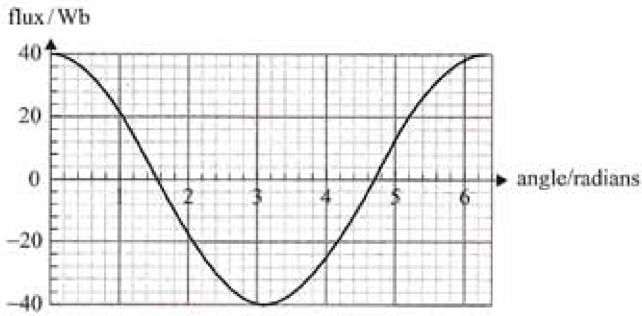

Figure 8.12 For question 8.