Stellar radiation

This chapter introduces two main tools in the study of stellar structure: the black-body radiation law and the related Wien displacement law. The significance of stellar spectra is discussed as well as the properties of a few special star systems, such as binary stars. The chapter closes with a discussion of another major tool in astrophysics: the Hertzsprung–Russell diagram.

Objectives

By the end of this chapter you should be able to:

- understand that a star is in equilibrium under the action of two opposing forces, *gravitation* and the *radiation pressure of the star*;
- appreciate that *nuclear fusion* provides the energy source of a star;
- give the definitions of *luminosity*, $L = \sigma A T^4$, as the power radiated into space by a star and *apparent brightness*, $b = \frac{L}{4\pi d^2}$, as the power received per unit area on earth;
- state the *Wien displacement law*, $\lambda_0 T = 2.90 \times 10^{-3}$ K m, and solve problems using it;
- appreciate the kind of information a *stellar spectrum* can provide;
- state the main properties of *main sequence stars*, *red giants*, *white dwarfs* and *binary stars*;
- describe the structure of an *HR diagram* and place the main types of stars on the diagram.

The energy source of stars

A star such as our own sun radiates an enormous amount of energy into space – about 10^{26} J s^{-1}. The source of this energy is nuclear fusion in the interior of the star, in which nuclei of hydrogen fuse to produce helium and release energy in the process. Because of the *high temperatures* in the interior of the star, the electrostatic repulsion between protons can be overcome and hydrogen nuclei can fuse. Because of the *high pressure* in stellar interiors, the nuclei are sufficiently close to each other to give a high probability of collision and hence fusion. The sequence of nuclear fusion reactions that take place is called the *proton–proton cycle* and consists of

$$^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + {}^0_1\text{e}^+ + {}^0_0\nu_e$$
$$^1_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + {}^0_0\gamma$$
$$^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2{}^1_1\text{H}$$

Energy is released at each stage of the cycle but most of it is released in the third and final stage. The energy produced is carried away by the photons and neutrinos produced in the reactions. As these particles move outwards they collide with surrounding protons and electrons and give them some of the energy. Thus, gradually, most of the particles in the star will receive some of the kinetic energy produced. The motion of the particles inside the star, as a result of the energy they receive, can stabilize the star against gravitational collapse. Note that the net effect of these reactions is to turn four hydrogen nuclei into one helium:

$$4{}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + 2e^{+} + 2\nu_e + 2\gamma$$

Helium, being heavier than hydrogen, collects in the core of the star. The energy released in the reactions above can be calculated by the methods of nuclear physics we studied earlier. The energy released per reaction is 26.7 MeV or 3.98×10^{-12} J.

▶ Nuclear fusion provides the energy that is needed to keep the star hot, so that the radiation pressure is high enough to oppose further gravitational contraction, and at the same time to provide the energy that the star is radiating into space (see Figure E2.1).

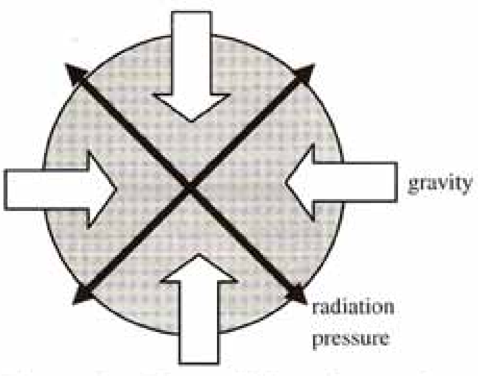

Figure E2.1 The stability of a star depends on equilibrium between two opposing forces: gravitation, which tends to collapse the star, and radiation pressure, which tends to make it expand.

Luminosity

▶ Luminosity is the amount of energy radiated by the star per second; that is, it is the power radiated by the star. As shown in the next section, luminosity depends on the surface temperature and surface area of the star.

Consider a star of luminosity L. Imagine a sphere of radius d centred at the location of the star. If the star is assumed to radiate uniformly in all directions, then the energy radiated in 1 s can be thought to be distributed over the surface of this imaginary sphere. A detector of area a placed somewhere on this sphere will receive a small fraction of this total energy (see Figure E2.2a). The fraction is equal to the ratio of the detector area a to the total surface area of the sphere; that is, the received energy per second will be $\frac{aL}{4\pi d^2}$.

▶ The received energy per second per unit area of detector is called the apparent brightness and is given by

$$b = \frac{L}{4\pi d^2}$$

The units of apparent brightness are W m^{-2}.

This shows that the apparent brightness is directly proportional to the intrinsic luminosity, and varies as the inverse square of the star's distance (see Figure E2.2b).

Apparent brightness is measured using a *charge-coupled device* (CCD), which offers many advantages over the conventional photographic film method (see Chapter 8.2). A CCD has a photosensitive silicon surface that releases an electron when it is hit by a photon. The number of electrons released is proportional to the number of photons that hit the surface. Thus, the amount of charge is a direct measure of the brightness of the object being observed. The

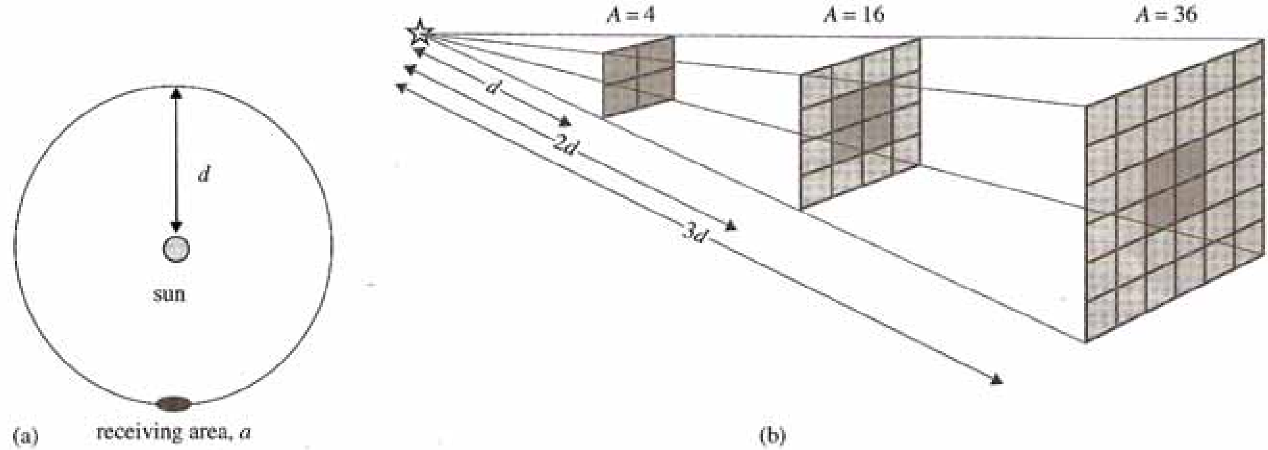

Figure E2.2 (a) The sun's energy is distributed over an imaginary sphere of radius equal to the distance between the sun and the observer. The observer thus receives only a very small fraction of the total energy, equal to the ratio of the receiver's area to the total area of the imaginary sphere. (b) The inverse square law:

$$\text{observed brightness} \propto \frac{1}{\text{area } A} \propto \frac{1}{(\text{distance } d)^2}$$

silicon surface is divided into many smaller areas, called *pixels*, and the charge released in each pixel can then be used (with digital techniques) to reconstruct an image of the object being observed. CCDs are more than 50 times more efficient in recording the photons arriving at the device than conventional photographic film.

Black-body radiation

A body of surface area *A* and *absolute* temperature *T* radiates energy away in the form of electromagnetic waves, according to the Stefan–Boltzmann law.

▶ The amount of energy per second radiated by a star of surface area A and absolute *surface* temperature T (i.e. the *luminosity*) is given by

$$L = \sigma A T^4$$

where the constant σ is called the Stefan–Boltzmann constant ($\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}).

If we now recall the definition of apparent brightness given in the previous section, we see that

$$b = \frac{\sigma A T^4}{4\pi d^2}$$

Example questions

Q1

The radius of star A is three times that of star B and its temperature is double that of B. Find the ratio of the luminosity of A to that of B.

Answer

$$\frac{L_A}{L_B} = \frac{\sigma 4\pi (R_A)^2 T_A^4}{\sigma 4\pi (R_B)^2 T_B^4}$$
$$= \frac{(R_A)^2 T_A^4}{(R_B)^2 T_B^4}$$
$$= \frac{(3R_B)^2 (2T_B)^4}{(R_B)^2 T_B^4}$$
$$= 3^2 \times 2^4 = 144$$

Q2

The stars in Example question 1 have the same apparent brightness when viewed from earth. Calculate the ratio of their distances.

Answer

$$\frac{b_A}{b_B} = 1$$
$$= \frac{L_A / (4\pi d_A^2)}{L_B / (4\pi d_B^2)}$$

$$= \frac{L_A}{L_B}\frac{d_B^2}{d_A^2}$$

$$= 144\frac{d_B^2}{d_A^2}$$

$$\Rightarrow \frac{d_A}{d_B} = 12$$

Q3

The apparent brightness of a star is 6.4×10^{-8} W m^{-2}. If its distance is 15 ly, what is its luminosity?

Answer

We use $b = \frac{L}{4\pi d^2}$ to find

$$L = b4\pi d^2$$

$$= \left(6.4 \times 10^{-8}\frac{\text{W}}{\text{m}^2}\right) \times 4\pi \times (15 \times 9.46 \times 10^{15})^2\ \text{m}^2$$

$$= 1.62 \times 10^{28}\ \text{W}$$

Q4

A star has half the sun's surface temperature and 400 times its luminosity. How many times bigger is it?

Answer

We have that

$$400 = \frac{L}{L_{sun}}$$

$$= \frac{\sigma 4\pi (R)^2 T^4}{\sigma 4\pi (R_{sun})^2 T_{sun}^4}$$

$$= \frac{(R)^2 (T_{sun}/2)^4}{(R_{sun})^2 T_{sun}^4}$$

$$= \frac{R^2}{(R_{sun})^2 16}$$

$$\frac{R^2}{(R_{sun})^2 16} = 400$$

$$\Rightarrow \frac{R^2}{(R_{sun})^2} = 16 \times 400$$

$$\Rightarrow \frac{R}{R_{sun}} = 80$$

The energy radiated by a star is in the form of electromagnetic radiation and is distributed over an infinite range of wavelengths. Figure E2.3 shows what is called the spectrum of a black body, that is, the energy radiated per second per wavelength interval from a unit area of the body. The horizontal axis represents wavelength in micrometres. The vertical scale has units of W m^{-3}.

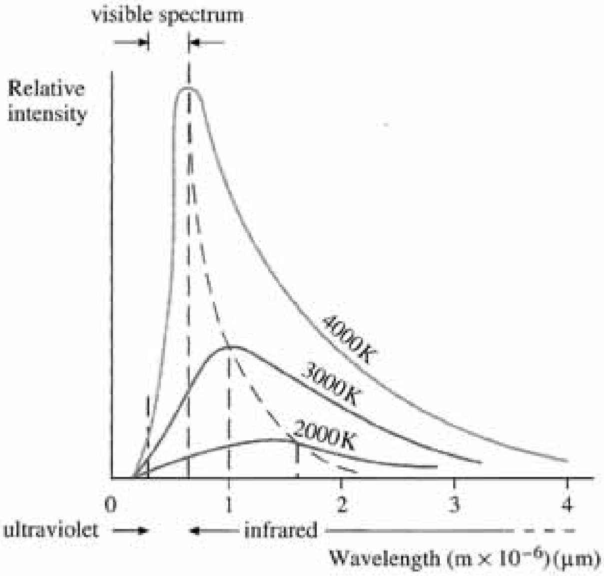

Figure E2.3 Radiation profiles at different temperatures. The broken lines show how the peak intensity, and the wavelength at which this occurs, vary with temperature. The overall intensity is represented by the area under the graph.

Most of the energy is emitted around the peak wavelength. Calling this wavelength λ_0, we see that the colour of the star is mainly determined by the colour corresponding to λ_0. The area under the black-body curve is the total power radiated from a unit area, irrespective of wavelength, and is thus given by σT^4.

▶ The Wien displacement law relates the wavelength λ_0 to *surface temperature* T:

$$\lambda_0 T = \text{constant} = 2.90 \times 10^{-3}\ \text{K m}$$

which implies that the higher the temperature, the lower the wavelength at which most of the energy is radiated.

Example questions

Q5

The sun has an approximate black-body spectrum with most of the energy radiated at a wavelength of 5.0×10^{-7} m. Find the surface temperature of the sun.

Answer

From Wien's law

$5.0 \times 10^{-7}\ \text{m} \times T = 2.9 \times 10^{-3}\ \text{K m}$

that is

$T = 5800\ \text{K}$

Q6

The sun (radius $R = 7.0 \times 10^{8}$ m) radiates a total power of 3.9×10^{26} W. Find its surface temperature.

Answer

From $L = \sigma A T^4$ and $A = 4\pi R^2$ we find

$$T = \left(\frac{L}{\sigma 4\pi R^2}\right)^{1/4} \text{K} \approx 5800\ \text{K}$$

Stellar spectra

A great wealth of information can be gathered about a star from studies of its spectrum.

Temperature

The surface temperature of the star is determined by measuring the wavelength at which most of the radiation is emitted (see Figure E2.4).

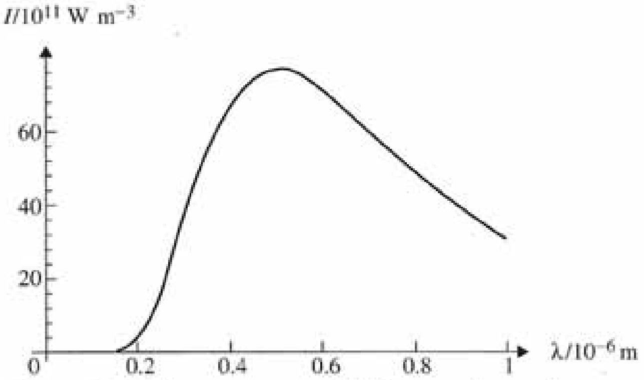

Figure E2.4 The spectrum of this star shows that most of the energy is emitted at a wavelength of about $\lambda = 5 \times 10^{-7}$ m. Use of Wien's law then allows the determination of the surface temperature of the star.

For the star of Figure E2.4, Wien's law gives

$$\lambda_0 T = 2.90 \times 10^{-3}\ \text{K m}$$
$$\Rightarrow T = \frac{2.90 \times 10^{-3}}{5 \times 10^{-7}\ \text{K}}$$
$$= 5800\ \text{K}$$

Chemical composition

In practice it is not always possible to obtain a spectrum like that of Figure E2.4. It is much more common to obtain an absorption spectrum in which dark lines are seen superimposed on a background of continuous colour (shown in black and white in Figure E2.5). Each dark line represents the absorption of light of a specific frequency by a specific chemical element in the star's atmosphere.

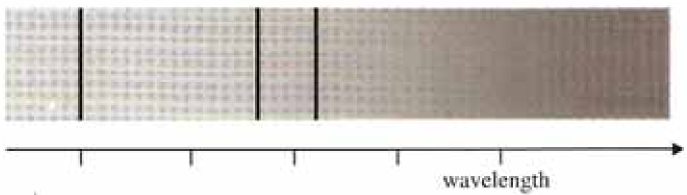

Figure E2.5 Absorption spectrum of a star showing three absorption lines. A real spectrum would show thousands of dark lines.

It has been found, however, that most stars have essentially the same chemical composition, yet show different absorption spectra. The reason for this difference is that different stars have different temperatures. Consider two stars with the same content of hydrogen. One is hot, about 25 000 K, and the other cool, about 10 000 K. The hydrogen in the hot star is ionized, which means the electrons have left the hydrogen atoms. These atoms cannot absorb any light passing through them, since there are no bound electrons that can absorb the photons and make transitions to higher energy states. Thus, the hot star will not show any absorption lines at hydrogen wavelengths. The cooler star, however, has many of its hydrogen atoms in the energy state $n = 2$. Electrons in this state can absorb photons to make transitions to states such as $n = 3$ and $n = 4$, giving rise to the characteristic hydrogen absorption lines. Similarly, an even cooler star of temperature, say, 3000 K will have most of its electrons in the ground state of hydrogen atoms and so can only absorb photons corresponding to ultraviolet wavelengths. These will not result in dark lines in an optical spectrum.

In this way, study of absorption spectra gives information about the temperature of the star and its chemical composition. Of course, as discussed in the last paragraph, the understanding is that absence of certain lines does not necessarily imply the absence of the corresponding chemical element.

Stars are divided into seven *spectral classes* according to their colour (see Table E2.1). As we have just seen, colour is related to surface temperature. The spectral classes are called O, B, A, F, G, K and M. (Remembered as Oh Be A Fine Girl/Guy Kiss Me!)

Spectral class	Colour	Temperature/K
O	Blue	25000–50000
B	Blue-white	12000–25000
A	White	7500–12000
F	Yellow-white	6000–7500
G	Yellow	4500–6000
K	Yellow-red	3000–4500
M	Red	2000–3000

Table E2.1 Colour and temperature characteristics of spectral classes.

It is known from spectral studies that hydrogen is the predominant element in normal main sequence stars (see next section), making up to 70% of their mass, followed by helium with 28%; the rest is made up of heavier elements.

Radial velocity

If a star moves away from or toward us, its spectral lines will show a Doppler shift. The shift will be toward the red if the star moves away, and toward the blue if it comes toward us. Measurement of the shift allows the determination of the radial velocity of the star.

Rotation

If a star rotates, then part of the star is moving toward the observer and part away from the observer. Thus, light from the different parts of the star will again show Doppler shifts, from which the rotation speed may be determined.

Magnetic fields

In a magnetic field a spectral line may split into two or more lines (the Zeeman effect). Measurement of the amount of splitting yields information on the magnetic field of the star.

The Hertzsprung–Russell diagram

Astronomers realized early on that there was a correlation between the luminosity of a star and its surface temperature. The higher the temperature, the higher the luminosity. In the early part of the twentieth century, the Danish astronomer, Ejnar Hertzsprung, and the American, Henry Norris Russell, independently pioneered plots of stellar luminosities. Hertzsprung plotted luminosities versus surface temperature and Russell plotted absolute magnitude versus spectral class. Such plots are now called Hertzsprung–Russell (HR) diagrams. In the HR diagram that follows (Figure E2.6), the vertical axis represents luminosity in units of the sun's luminosity (i.e. 1 on the vertical axis corresponds to the solar luminosity of 3.9×10^{26} W). The horizontal axis shows the surface temperature of the star (in thousands of kelvin). The temperature *decreases as we move to the right*. Also shown at the top of the diagram is the *spectral class* for each star, which is an alternative way to label the horizontal axis. The luminosity in this diagram varies from 10^{-5} to 10^{5}, a full 10 orders of magnitude, whereas the temperature varies from 3000 K to 25 000 K. For this reason, the scale on the axes is *not* linear.

As more and more stars were placed on the HR diagram, it became clear that a pattern was emerging. The stars were not randomly distributed on the diagram.

▶ Three clear features emerge from the HR diagram.

1 Most stars fall on a strip extending diagonally across the diagram from top left to bottom right. This is called the main sequence.
2 Some large stars, reddish in colour, occupy the top right – these are the *red giants* (large, cool stars).
3 The bottom left is a region of small stars known as *white dwarfs* (small and hot).

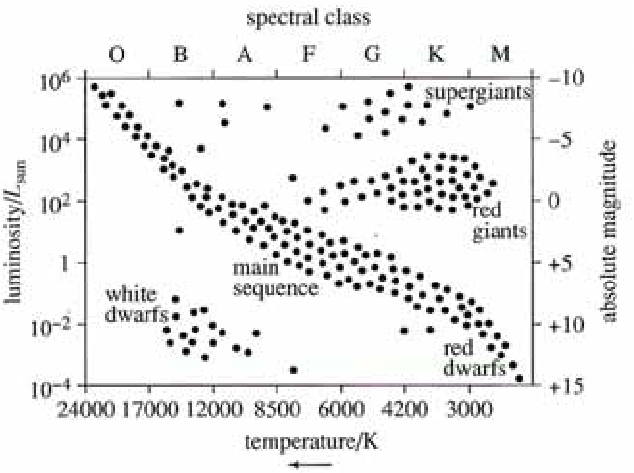

Figure E2.6 The Hertzsprung–Russell diagram. The (surface) temperature increases to the left. Note that the scales are not linear.

In fact, about 90% of all stars are main sequence stars, 9% are white dwarfs and 1% are red giants. Another feature of the HR diagram is that, as we move along the *main sequence* toward hotter stars, the mass of the stars increases as well. Thus, the right end of the main sequence is occupied by red dwarfs and the left by blue giants. Note that as we move up the main sequence (right to left), the mass of the stars increases. .

Note that, once we know the temperature of a star (for example, through its spectrum), the HR diagram can tell us the luminosity of the star with an acceptable degree of accuracy, provided it is a main sequence star. (The main sequence is, after all, not a mathematical line but a broad band.) This observation is the basis for a method that determines the distance to a star called spectroscopic parallax (see Option E3).

Types of stars

As we have just seen, the HR diagram makes a clear division of stars into various types.

Main sequence stars

Our sun is a typical member of the main sequence. It has a mass of 2×10^{30} kg, a radius of 7×10^8 m, an average density of 1.4×10^3 kg m^{-3} and radiates at a rate of 3.9×10^{26} W. What distinguishes different main sequence stars is their mass. Main sequence stars produce enough energy in their core, from nuclear fusion of hydrogen into helium, to exactly counterbalance the tendency of the star to collapse under its own weight. The luminosity of stars on the main sequence increases as the mass increases.

Red giants

Red giants are another important class of stars. They are very large, cool stars with a reddish appearance. The luminosity of red giants is considerably greater than the luminosity of main sequence stars of the same temperature; they can, in fact, be a million or even a billion times bigger. Treating them as black bodies radiating according to the Stefan–Boltzmann law means that a luminosity of 10^6 times bigger corresponds to an area of 10^6 times bigger, which means a radius of 10^3 times bigger. This explains the name given to these stars. The mass of a red giant can be as much as 1000 times the mass of our sun, but their huge size also implies small densities. In fact, a red giant will have a central hot core surrounded by an enormous envelope of extremely tenuous gas.

White dwarfs

These are very common stars but their faintness makes them hard to detect. A well-known white dwarf is Sirius B, the second star in a binary star system (double star), the other member of which, Sirius A, is the brightest star in the evening sky. Sirius A and B have about the same surface temperature (about 10 000 K) but the luminosity of Sirius B is about 10 000 times smaller. This means that Sirius B has a radius that is 100 times smaller than that of Sirius A. Here is a star of mass roughly that of the sun with a size similar to that of the earth. This means that its density is about 10^6 times the density of the earth!

White dwarfs form when a star collapsing under its own gravitation stabilizes as a result of *electron degeneracy pressure*. This means that the electrons of the star are forced into the same quantum states. To avoid that, the Pauli exclusion principle forces them to acquire large kinetic energies. The large electron energies can then withstand the gravitational pressure of the star.

In addition, we may identify three other important star types.

Variable stars

Whereas the luminosity of our sun and other main sequence stars has remained constant over millions of years, stars exist that show a variation in their luminosity with time. These are called *variable stars*. The variation of luminosity with time (a graph showing the variation of luminosity with time is known as the *light curve* of the star) can be periodic or non-periodic. The reasons for the variable luminosity are mainly changes in the internal structure of the star. For example, a normal main sequence star will, as part of its evolutionary process, grow in size as its outer envelope expands. In doing so, it may eject mass from the outer layers, forming what is called a *planetary nebula*, with an ensuing increase in the star's luminosity. Similarly, if the star is substantially heavier than the sun, the release of mass and energy from the outer envelope is even more dramatic, resulting in a *supernova* with luminosity increases by factors of a million. In the case of binary stars (see later), matter can be transferred from one star to the other and, on being heated, this matter can radiate, again increasing the star's luminosity.

Cepheids

Most prominent among the class of *periodic* variables are the Cepheid stars, because there exists a relationship between the period of the light curve and the peak luminosity of these stars. Thus, observation of a Cepheid over time allows the determination of its period and hence its peak luminosity. Knowledge of the luminosity is important since comparison with the apparent brightness yields the distance of the star. Cepheids have periods from 1 to 50 days.

The study of variable stars is important since it provides much information about the internal structure of the star and is a testing ground for theories about stellar structure.

Binary stars

A system of two stars that orbit a common centre is called a binary star system. Depending on the method used to observe them, binaries fall into three classes:

- visual
- eclipsing
- spectroscopic.

Binaries are important because they allow for the determination of stellar masses as explained below.

Visual binaries – These appear as two separate stars when viewed through a telescope. They are in orbit around a common centre, the centre of mass of the two stars, as shown in Figure E2.7 in the simplified case of circular orbits.

It is shown in the chapter on gravitation that the common period of rotation for a binary is given by

$$T^2 = \frac{4\pi^2 d^3}{G(M_1 + M_2)}$$

where d is the distance between the two stars. Thus we can conclude that:

▶ Measurement of the separation distance and period gives the *sum* of the masses making up the binary.

To determine the masses individually, we need information about the orbit of each star. Note that the inner star is the more massive of the two.

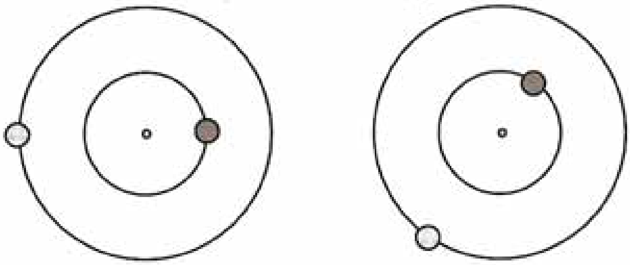

Figure E2.7 A binary star system. The stars rotate about their centre of mass. The two stars are always diametrically opposite each other.

Example question

Q7

A visual binary with a period of 50 yr is at a distance from earth of 8.79 ly. The distance between the stars subtends an angle at earth (the 'angular diameter') of 7.56 arcseconds. Find the sum of the masses in the binary.

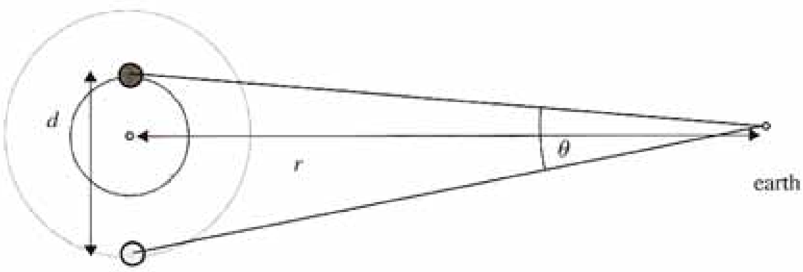

Figure E2.8 (Not to scale).

Answer

See Figure E2.8.

Figure E2.8 shows what is meant by 'angular diameter' – it is the angle θ that the separation of the stars subtends at earth. The distance separating the stars is $d = r\theta$ where r is the distance to the binary. The angle must be expressed in radians, that is (" = arcseconds)

$$\theta = 7.56'' = \frac{7.56^\circ}{3600} = \frac{7.56^\circ}{3600} \times \frac{\pi}{180^\circ} = 3.665 \times 10^{-5}\ \text{rad}$$

The distance to the binary in metres is

$$r = 8.79 \times 9.46 \times 10^{15} = 8.31 \times 10^{16}\ \text{m}$$

Hence the separation of the stars is $d = 3.05 \times 10^{12}$ m. From the formula for the period

$$M_1 + M_2 = \frac{4\pi^2 d^3}{GT^2} = \frac{4\pi^2(3.05 \times 10^{12})^3}{6.67 \times 10^{-11} \times (50 \times 365 \times 24 \times 60 \times 60)^2} = 6.75 \times 10^{30}\ \text{kg}$$

or 3.4 solar masses.

Eclipsing binaries – If the plane of the orbit of the two stars is suitably oriented relative to that of the earth, the light from one of the stars in the binary may be blocked by the other, resulting in an eclipse of the star, which may be total or partial. If a bright star (light grey circle) is orbited by a dimmer companion (dark circle), the light curve has the pattern shown in Figure E2.9. Such an example is provided by the system of AR Cassiopeia.

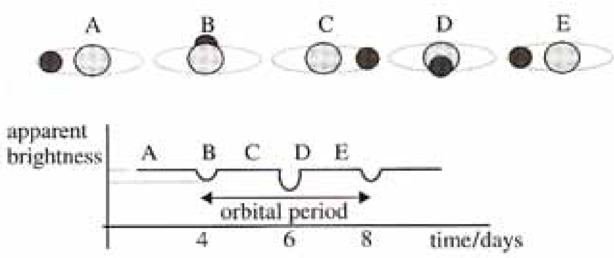

Figure E2.9 The light curve of AR Cassiopeia shows dips in brightness as the dimmer companion disappears behind the brighter star. When the dim star is in front, the dip is the largest.

Example question

Q8

Discuss the light curve of the eclipsing binary system Algol shown in Figure E2.10.

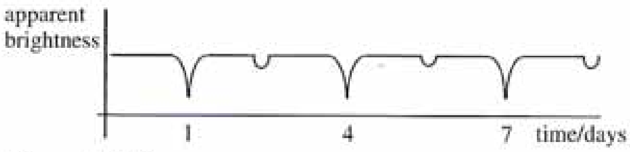

Figure E2.10.

Answer

This is an example of an eclipsing binary system in which the brighter of the two stars partially disappears from view every 3 days. The large dip in brightness occurs when the brighter star is behind the dimmer one. The small dip in brightness occurs when the brighter star is in front of the dimmer star.

Spectroscopic binaries – This system is detected by analysing the light from one or both of its members and observing that there is a periodic Doppler shifting of the lines in the spectrum. A blueshift is expected as the star approaches the earth and a redshift as it moves away from the earth in its orbit around its companion (Figure E2.11).

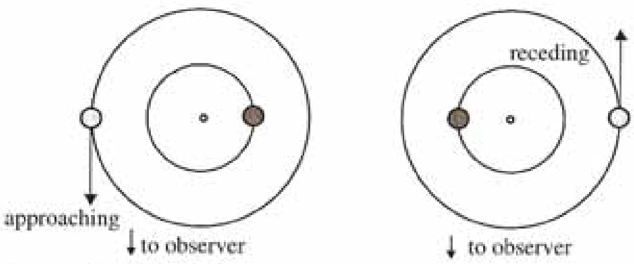

Figure E2.11 A binary star system. The stars rotate about their centre of mass. If the light grey star is the brighter, its light arrives at an observer blueshifted in the first diagram and redshifted in the second. In both positions shown, the stars are said to be in conjunction.

If λ_0 is the wavelength of a spectral line and λ the wavelength received on earth, the shift, z, of the star is defined as

$$z = \left| \frac{\lambda - \lambda_0}{\lambda_0} \right|$$

If the speed of the source is small compared with the speed of light, it can be shown that

$$z = \frac{v}{c}$$

which shows that the shift is indeed directly proportional to the source's speed.

The top diagram in Figure E2.12 shows a spectrum with one line and the other two diagrams show what this line would look like if it were blue- or redshifted.

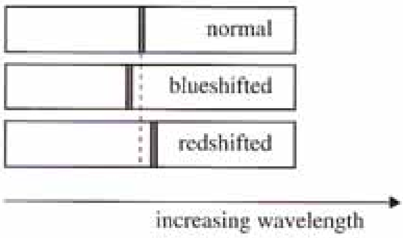

Figure E2.12 A normal spectral line (observed when the star's velocity is normal to the line of sight) is periodically blue- and redshifted as the star revolves in its orbit. In this example we are assuming that the second star is very dim so its light is not recorded. See questions 23 and 24 at the end of the chapter for the case in which light from both stars is analysed.

There is more discussion of the Doppler effect for light in Chapter 4.5 and on page 536.

Example question

Q9

The blueshifts and redshifts of the bright star in Figure E2.12 are 3.4×10^{-5}. If it is known that the two stars are equal in mass and the distance separating them is 2.8×10^{12} m, what are these masses?

Answer

From the Doppler formula, $v = zc = 10\,200\ \text{m s}^{-1}$. From

$$v^2 = \frac{GM_1^{\,2}}{d(M_1 + M_2)}$$
$$= \frac{GM}{2d}$$

it follows that

$$M = \frac{2v^2 d}{G}$$
$$= 4.4 \times 10^{30}\ \text{kg}$$

Questions

1 The light from a star a distance of 70 ly away is received on earth with an apparent brightness of $3.0 \times 10^{-8}\ \text{W m}^{-2}$. Calculate the luminosity of the star.

2 The luminosity of a star is 4.5×10^{28} W and its distance from earth is 88 ly. Calculate the apparent brightness of the star.

3 The apparent brightness of a star is $8.4 \times 10^{-10}\ \text{W m}^{-2}$ and its luminosity 6.2×10^{32} W. Calculate the distance to the star in light years.

4 Two stars have the same size but one has a temperature that is four times larger.
(a) How much more energy per second does the hot star radiate?
(b) The apparent brightness of the two stars is the same; what is the ratio of the distance of the cooler star to that of the hotter star?

5 Two stars are the same distance from earth and their apparent brightnesses are $9.0 \times 10^{-12}\ \text{W m}^{-2}$ (star A) and $3.0 \times 10^{-13}\ \text{W m}^{-2}$ (star B). Calculate the ratio of the luminosity of star A to that of star B.

6 Take the surface temperature of our sun to be 6000 K and its luminosity to be 3.9×10^{26} W. Find, in terms of the solar radius, the radius of a star with:
(a) temperature 4000 K and luminosity 5.2×10^{28} W;
(b) temperature 9250 K and luminosity 4.7×10^{27} W.

7 Two stars have the same luminosity. Star A has a surface temperature of 5000 K and star B a temperature of 10 000 K.
(a) Which is the larger star and by how much?
(b) If the apparent brightness of A is double that of B, what is the ratio of the distance of A to that of B?

8 Star A has apparent brightness $8.0 \times 10^{-13}\ \text{W m}^{-2}$ and its distance is 120 ly. Star B has apparent brightness $2.0 \times 10^{-15}\ \text{W m}^{-2}$ and its distance is 150 ly. The two stars have the same size. Calculate the ratio of the temperature of star A to that of star B.

9 Two stars A and B emit most of their light at wavelengths of 650 nm and 480 nm respectively. If it is known that star A has twice the radius of star B, find the ratio of the luminosities of the stars.

10 Explain how the surface temperature of a star determines the spectral class to which it belongs.

11 Describe how the colour of the light from a star can be used to determine the surface temperature of the star.

12 Explain why a star on the top left of the main sequence will spend much less time on the main sequence than another star on the lower right.

13 Describe the main features of the HR diagram. What quantities can be plotted on the vertical axis and which on the horizontal? Why are the scales non-linear?

14 Describe how a stellar absorption spectrum is formed.

15 Figure E2.13 shows the intensity of a particular spectral line emitted by a non-rotating star. On the same graph, draw what you would expect if the star were rotating.

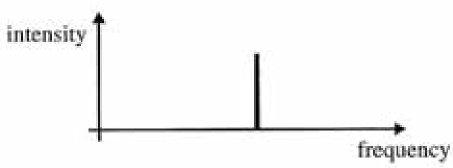

Figure E2.13 For question 15.

16 Show that, if the stars in a binary star system have the same mass, they share the same orbit.

17 Make a sketch of the light curve of an eclipsing binary of period 20 yr in which:
(a) both members are equally bright;
(b) the inner star is much brighter than the other.

(Assume that the line of sight is in the orbital plane.) In each case draw diagrams to show the relative position of the two stars for significant times during the period.

18 From redshift measurements in a spectroscopic binary, it is known that the ratio of the masses is 1.20. If the period of the binary is 40 yr, and from parallax measurements it is known that the two stars are separated by a distance of 2.4×10^{12} m, find the individual masses of each star in the binary.

19 A visual binary system is at a distance of 5.0 pc. The distance between the two stars subtends an angle of 4.5 arcseconds.

(a) What is this distance?

(b) The period of the binary is 87.8 yr. What is the sum of the masses of the stars making up the binary?

(c) The radius of the orbit of one of the stars subtends an angle of 1.91 arcseconds. What is the mass of each of the stars?

20 Describe what is meant by the term *white dwarf*. List two properties of the star. How does a white dwarf differ from a main sequence star of the same surface temperature?

21 A white dwarf, of mass half that of the sun and radius equal to one earth radius, is formed. What is the density of this white dwarf?

22 Where on the HR diagram would our sun lie at the time of its creation?

23 Figure E2.14 shows the spectrum of a spectroscopic binary.

(a) Explain the structure of this spectrum.

(b) Explain how it can be deduced that the stars are not equally massive.

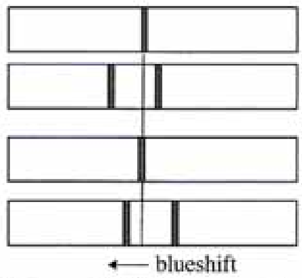

Figure E2.14 For questions 23 and 24.

(c) Show by appropriate diagrams the relative positions of the two stars that give rise to each of the four spectra shown.

24 For the binary star system described in question 23, assume that the redshift in the second diagram of Figure E2.14 is 3.4×10^{-5} and the blueshift is 4.7×10^{-5}. Find the ratio of the masses of the two stars in the binary.

25 A binary star system consists of two stars that have a ratio of apparent brightness equal to 10. Explain carefully how we can deduce that the ratio of the luminosities of the stars is also 10.

26 (a) Find the temperature of a star whose spectrum is shown in Figure E2.15.

(b) Assuming this is a main sequence star, what do you *estimate* its luminosity to be?

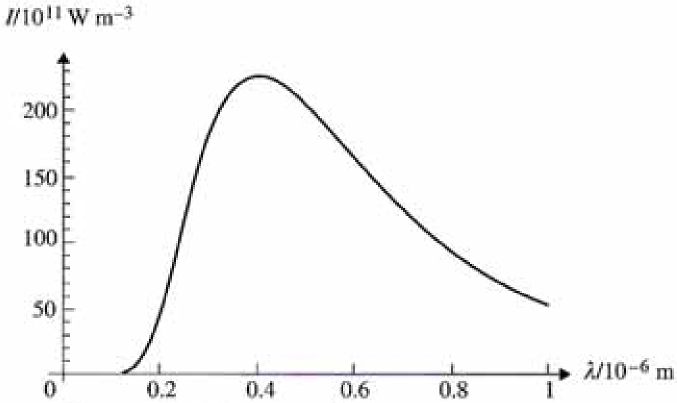

Figure E2.15 For question 26.