Stellar radiation

Reminder:

The **luminosity** of a star is related to its temperature T and its surface area A according to the equation:

$$\boxed{L(\mathrm{W}) = \sigma \cdot A \cdot T^4}$$

σ is a constant called Stefan-Boltzmann constant: $\sigma = 5{,}67 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$

The **apparent brightness** is related to the distance of observation d from the star:

$$\boxed{b(\mathrm{W \cdot m^{-2}}) = \frac{L}{4\pi \cdot d^2}}$$

A light year ly is the distance covered by light during one year of propagation.

$$1\ \mathrm{l.y} = 3.00 \times 10^8 \times 365 \times 24 \times 3600 = 9.46 \times 10^{15}\ \mathrm{m}$$

Wien's law relates the temperature of a black body to the wavelength for which there is a maximum of emission of light:

$$\boxed{\lambda_{\mathrm{max}} \cdot T = 2.9 \times 10^{-3}\ \mathrm{m \cdot K}}$$

1 The light from a star a distance of 70 ly away is received on earth with an apparent brightness of $3.0 \times 10^{-8}\ \mathrm{W\,m^{-2}}$. Calculate the luminosity of the star.

$$\boxed{L = 4\pi \cdot d^2 \cdot b}$$

$$L = 4\pi \cdot (70 \times 9.46 \times 10^{15})^2 \cdot 3.0 \times 10^{-8} = 1.7 \times 10^{29}\mathrm{W}$$

2 The luminosity of a star is 4.5×10^{28} W and its distance from earth is 88 ly. Calculate the apparent brightness of the star.

$$\boxed{b = \frac{L}{4\pi \cdot d^2}}$$

$$b = \frac{4.5 \times 10^{28}}{4\pi \cdot (88 \times 9.46 \times 10^{15})^2} = 5.2 \times 10^{-9}\ \mathrm{W \cdot m^{-2}}$$

3 The apparent brightness of a star is $8.4 \times 10^{-10}\ \mathrm{W\,m^{-2}}$ and its luminosity 6.2×10^{32} W. Calculate the distance to the star in light years.

$$\boxed{d = \sqrt{\frac{L}{4\pi \cdot b}}}$$

$$d = \sqrt{\frac{6.2 \times 10^{32}}{4\pi \cdot 8.4 \times 10^{-10}}} = \sqrt{\frac{6.2}{4\pi \cdot 8.4}} \times 10^{21} = 2.4 \times 10^{20}\ \mathrm{m} = 2.6 \times 10^{4}\ \mathrm{l.y}$$

4 Two stars have the same size but one has a temperature that is four times larger.

(a) How much more energy per second does the hot star radiate?

(b) The apparent brightness of the two stars is the same; what is the ratio of the distance of the cooler star to that of the hotter star?

a) Say the hotter star has a temperature T_2 and the cooler star a temperature T_1 :

$$T_2 = 4 \cdot T_1$$

The two stars have the same size:

$$A_2 = A_1$$

$$L(\mathrm{W}) = \sigma \cdot A \cdot T^4$$

Luminosity is proportional to the fourth power of the temperature.
Therefore:

$$L_2 = 4^4 \times L_1 = 2^8 \times L_1 = 256 \times L_1$$

b) Apparent brightness is inversely proportional to the square of the distance.

$$b_2 = \frac{L_2}{4\pi \cdot {d_2}^2} = b_1 = \frac{L_1}{4\pi \cdot {d_1}^2}$$

Because the two stars have the same brightness although star 2 has a bigger luminosity; it means that star 2 must be more distant than star 1.

Therefore:

$$\frac{L_2}{{d_2}^2} = \frac{L_1}{{d_1}^2}$$

$$\frac{d_1}{d_2} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{1}{256}} = \frac{1}{16}$$

5 Two stars are the same distance from earth and their apparent brightnesses are $9.0 \times 10^{-12}\ \mathrm{W\,m^{-2}}$ (star A) and $3.0 \times 10^{-13}\ \mathrm{W\,m^{-2}}$ (star B). Calculate the ratio of the luminosity of star A to that of star B.

$$L = 4\pi \cdot d^2 \cdot b$$

$$\boxed{\frac{L(\mathrm{A})}{L(\mathrm{B})} = \frac{b(\mathrm{A}) \cdot d(\mathrm{A})^2}{b(\mathrm{B}) \cdot d(\mathrm{B})^2}\ (1)}$$

As the stars are at the same distance from the Earth:

$$\frac{L(\mathrm{A})}{L(\mathrm{B})} = \frac{b(\mathrm{A})}{b(\mathrm{B})} = \frac{9.0 \times 10^{-12}}{3.0 \times 10^{-13}} = 30$$

6 Take the surface temperature of our sun to be 6000 K and its luminosity to be 3.9×10^{26} W. Find, in terms of the solar radius, the radius of a star with:

(a) temperature 4000 K and luminosity 5.2×10^{28} W;

(b) temperature 9250 K and luminosity 4.7×10^{27} W.

$$L(\mathrm{Sun}) = \sigma \cdot \pi \cdot R(\mathrm{Sun})^2 \cdot T(\mathrm{Sun})^4$$
$$L(\mathrm{Star}) = \sigma \cdot \pi \cdot R(\mathrm{Star})^2 \cdot T(\mathrm{Star})^4$$

If we divide one equation over the other:

$$\frac{R(\text{Star})}{R(\text{Sun})} = \sqrt{\frac{L(\text{Star})}{L(\text{Sun})}} \cdot (\frac{T(\text{Sun})}{T(\text{Star})})^2$$

a)

$$\frac{R(\text{Star})}{R(\text{Sun})} = \sqrt{\frac{5.2 \times 10^{28}}{3.9 \times 10^{26}}} \cdot \left(\frac{6000}{4000}\right)^2 = 26$$

b)

$$\frac{R(\text{Star})}{R(\text{Sun})} = \sqrt{\frac{4.7 \times 10^{27}}{3.9 \times 10^{26}}} \cdot \left(\frac{6000}{9250}\right)^2 = 4.6$$

7 Two stars have the same luminosity. Star A has a surface temperature of 5000 K and star B a temperature of 10 000 K.
(a) Which is the larger star and by how much?
(b) If the apparent brightness of A is double that of B, what is the ratio of the distance of A to that of B?

$$L(\text{A}) = \sigma \cdot \pi \cdot R(\text{A})^2 \cdot T(\text{A})^4$$
$$L(\text{B}) = \sigma \cdot \pi \cdot R(\text{B})^2 \cdot T(\text{B})^4$$
$$\boxed{\frac{R(\text{B})}{R(\text{A})} = \sqrt{\frac{L(\text{B})}{L(\text{A})}} \cdot \left(\frac{T(\text{A})}{T(\text{B})}\right)^2 \quad (2)}$$

a) The two stars have the same luminosity, but star B is hotter than star A; we can conclude that star A is bigger than star B.

$$\frac{R(\text{B})}{R(\text{A})} = \sqrt{\frac{L(\text{B})}{L(\text{A})}} \cdot (\frac{T(\text{A})}{T(\text{B})})^2$$
$$\frac{R(B)}{R(A)} = (\frac{10000}{5000})^2 = 4$$

b)

$$b(\text{A}) = \frac{L(\text{A})}{4\pi \cdot d(\text{A})^2}$$
$$b(\text{B}) = \frac{L(\text{B})}{4\pi \cdot d(\text{B})^2}$$
$$\frac{d(\text{A})}{d(\text{B})} = \sqrt{\frac{b(\text{B})}{b(\text{A})} \cdot \frac{L(\text{A})}{L(\text{B})}} = \sqrt{\frac{1}{2}} = 0.71$$

8 Star A has apparent brightness 8.0×10^{-13} W m^{-2} and its distance is 120 ly. Star B has apparent brightness 2.0×10^{-15} W m^{-2} and its distance is 150 ly. The two stars have the same size. Calculate the ratio of the temperature of star A to that of star B.

$$\boxed{\frac{R(\text{B})}{R(\text{A})} = \sqrt{\frac{L(\text{B})}{L(\text{A})}} \cdot \left(\frac{T(\text{A})}{T(\text{B})}\right)^2 \quad (2)}$$

$$\boxed{\frac{L(\text{A})}{L(\text{B})} = \frac{b(\text{A}) \cdot d(\text{A})^2}{b(\text{B}) \cdot d(\text{B})^2} \ (1)}$$

As the stars have the same size:

$$\frac{T(\text{A})}{T(\text{B})} = (\frac{L(\text{B})}{L(\text{A})})^{\frac{1}{4}} = (\frac{b(\text{A})}{b(\text{B})})^{\frac{1}{4}} \cdot (\frac{d(\text{A})}{d(\text{B})})^{\frac{1}{2}}$$

$$\frac{T(\text{A})}{T(\text{B})} = (\frac{8.0 \times 10^{-13}}{2.0 \times 10^{-15}})^{\frac{1}{4}} \cdot \left(\frac{120}{150}\right)^{\frac{1}{2}} = 4$$

9 Two stars A and B emit most of their light at wavelengths of 650 nm and 480 nm respectively. If it is known that star A has twice the radius of star B, find the ratio of the luminosities of the stars.

$$\boxed{\lambda_{\max}(\text{A}) \cdot T(\text{A}) = \lambda_{\max}(\text{B}) \cdot T(\text{B}) \ (3)}$$

$$\frac{T(\text{A})}{T(\text{B})} = \frac{\lambda_{\max}(\text{B})}{\lambda_{\max}(\text{A})} < 1$$

From (2):

$$\frac{L(\text{B})}{L(\text{A})} = (\frac{R(\text{B})}{R(\text{A})})^2 \cdot (\frac{T(\text{B})}{T(\text{A})})^4$$

Therefore:

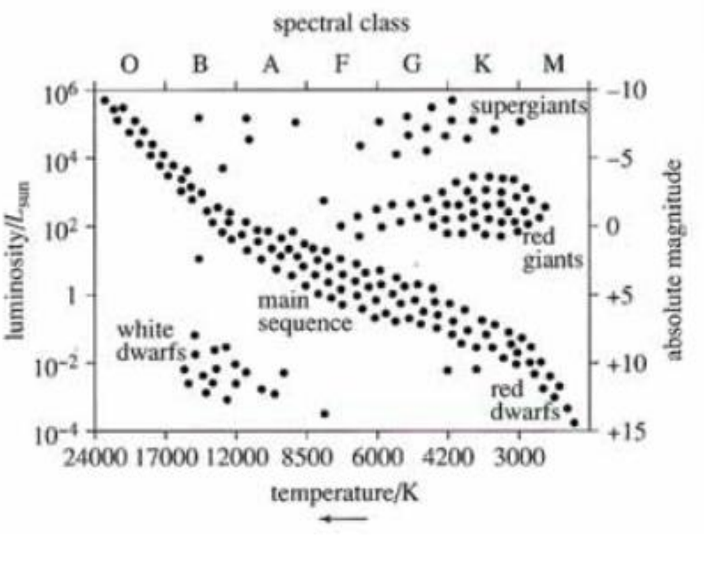
$$\frac{L(\text{B})}{L(\text{A})} = (\frac{R(\text{B})}{R(\text{A})})^2 \cdot (\frac{\lambda_{\max}(\text{A})}{\lambda_{\max}(\text{B})})^4$$

$$\frac{L(\text{B})}{L(\text{A})} = (\frac{1}{2})^2 \cdot \left(\frac{650}{480}\right)^4 = 0.84$$

Or

$$\frac{L(\text{A})}{L(\text{B})} = 1.2$$

10 Explain how the surface temperature of a star determines the spectral class to which it belongs.



As the Hertzsprung-Russel diagram shows above, there is a relationship between the surface temperature of a star and its spectral class.