Question 1

a. A black body is a body whose emissivity is equal to 1; it means that it absorbs all the radiation sent to it so that the radiation it emits results only from its temperature.
According to the Stefan-Boltzmann law:

$$P = e \cdot \sigma \cdot A \cdot T^4$$

$$\sigma = 5.68 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$$

Here: $e = 1$

b. A good approximation of a black body is charcoal.

c. If the temperature of the body is increased from 50°C ($T_1 = 323\mathrm{K}$) to 100°C ($T_2 = 373\mathrm{K}$) ; the power is increased by the rate:

$$\frac{P_2}{P_1} = \frac{{T_2}^4}{{T_1}^4} = (\frac{T_2}{T_1})^4 = (\frac{373}{323})^4 = \mathbf{1.8}$$

Question 2

a. The temperatures of the two bodies are equal, as the peak of the curve is reached for the same wavelength λ_{max} according to Wien's law that stipulates:

$$\lambda_{\mathrm{max}} \cdot T = 2.90 \times 10^{-3}\ \mathrm{m \cdot K}$$

b. The two bodies have got the same shape (same area) and the same temperature according to the previous question; one has a emissivity e_1 equal to 1 (the black body), the other an emissivity equal to e_2.

$$\frac{P_2}{P_1} = \frac{e_2}{e_1}$$

Therefore:

$$\frac{e_2}{1} = e_2 = \frac{(P_2)_{\mathrm{max}}}{(P_1)_{\mathrm{max}}} = \frac{1.1}{1.9} = \mathbf{0.6}$$

Question 3

The body with an emissivity e, an area A and a temperature T radiates a power:

$$P = \sigma \cdot e \cdot A \cdot T^4$$

$$T = (\frac{P}{\sigma \cdot A})^{\frac{1}{4}} = (\frac{1.35 \times 10^9}{0.8 \times 5 \times 10^6 \times 5.68 \times 10^{-8}})^{\frac{1}{4}} = (\frac{1.35 \times 10^{11}}{4 \times 5.68})^{\frac{1}{4}} = \mathbf{278\ K}$$

Question 4

a. The sun radiates a power:

$$P = \sigma_{\mathrm{Sun}} \cdot A \cdot {T_{\mathrm{Sun}}}^4$$

It is radiated uniformly through space; so that the intensity received at the distance d is:

$$I = \frac{P_{\mathrm{Sun}}}{4\pi \cdot d^2}$$

The effective power absorbed by the Earth is: $P_{\mathrm{absorbed}} = I \cdot \pi \cdot {R_E}^2$ where R_E is the radius of the Earth.

At equilibrium: $P_{\mathrm{absorbed}} = P_{\mathrm{emitted}} = \sigma_{\mathrm{Earth}} \cdot A_{\mathrm{Earth}} \cdot {T_{\mathrm{Earth}}}^4 = \sigma_{\mathrm{Earth}} \cdot 4\pi \cdot {R_E}^2 \cdot {T_{\mathrm{Earth}}}^4$

So that: $I \cdot \pi \cdot {R_E}^2 = \frac{P_{\mathrm{Sun}}}{4\pi \cdot d^2} \cdot \pi \cdot {R_E}^2 = \sigma_{\mathrm{Earth}} \cdot 4\pi \cdot {R_E}^2 \cdot {T_{\mathrm{Earth}}}^4$

$$T_{\text{Earth}} = \left(\frac{\frac{P_{\text{Sun}}}{4\pi \cdot d^2} \cdot \pi \cdot {R_E}^2}{\sigma_{\text{Earth}} \cdot 4\pi \cdot {R_E}^2}\right)^{\frac{1}{4}} = \left(\frac{P_{\text{Sun}}}{\sigma_{\text{Earth}} \cdot 16\pi \cdot d^2}\right)^{\frac{1}{4}} = \left(\frac{P_{\text{Sun}}}{\sigma_{\text{Earth}} \cdot 16\pi}\right)^{\frac{1}{4}} \cdot \left(\frac{1}{d}\right)^{\frac{1}{2}} = \frac{A}{\sqrt{d}}$$

This equation shows that the temperature of the Earth is inversely proportional to the square root of the distance that separates it from the Sun.

b. If the distance decreased by 1% (that means the distance is multiplied by 0.99) then the temperature of the Earth would be multiplied by a factor equal to $(\frac{1}{0.99})^{\frac{1}{2}} = 1.005$; with an initial temperature equal to 288 K, it would become 289.45 K so an increase of **1.4 K**.

Question 5

a. Intensity is power per unit of surface:

$$I(\mathrm{W \cdot m^{-2}}) = \frac{P(\mathrm{W})}{S(\mathrm{m^2})}$$

b. The power radiated by the source in the form of electromagnetic radiated is distributed uniformly over spheres centered on the source; the surface of a sphere of radius d is given by $4\pi \cdot d^2$; therefore, the intensity at a distance d is given by:

$$I(\mathrm{W \cdot m^{-2}}) = \frac{P(\mathrm{W})}{4\pi \cdot d^2}$$

The expression of the power emitted by a source of emissivity e and of temperature T is given by Stefan-Boltzmann law:

$$P = \sigma \cdot e \cdot A \cdot T^4$$

Therefore:

$$I(\mathrm{W \cdot m^{-2}}) = \frac{P(\mathrm{W})}{4\pi \cdot d^2} = \frac{\sigma \cdot e \cdot A \cdot T^4}{4\pi \cdot d^2}$$

$$\sigma = 5.68 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}\ ; A = 1.6\ \mathrm{m^2}\ ; T = 273 + 37 = 310\ \mathrm{K}\ ;\ \mathrm{e} = 0.9\ ; d = 5\ \mathrm{m}$$

$$I(\mathrm{W \cdot m^{-2}}) = \frac{\sigma \cdot e \cdot A \cdot T^4}{4\pi \cdot d^2} = \frac{5.68 \times 10^{-8} \times 0.9 \times 1.6 \times 310^4}{4\pi \times 5^2} = \mathbf{2.4\ W \cdot m^{-2}}$$

Question 6: Done in question 5

Question 7:

a. According to Wien's law:

$$\lambda_{\mathrm{max}} \cdot T = 2.90 \times 10^{-3}\ \mathrm{m \cdot K}$$

We read on the graph: $\lambda_{\mathrm{max}} = 0.65 \times 10^{-5}$ m.

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{\mathrm{max}}} = \frac{2.9 \times 10^{-3}}{0.65 \times 10^{-5}} = \mathbf{446\ K}$$

b. For a temperature of 600K, that is a higher temperature, the curve is overall higher and shifted to the left, towards the lowest wavelengths, that is towards the blue side of the electromagnetic spectrum.

Question 8: Albedo is defined as a ratio:

$$a = \frac{Reflected\ power}{Incident\ power}$$

The albedo depends on the shininess of the material, on its color, on its shape

Question 9:

According to the definition of the albedo:

$$a = \frac{Reflected\ power}{Incident\ power} = \frac{350 - 250}{350} = \mathbf{0.29}$$

The intensity of the radiation absorbed is : $I = \mathbf{250\ W \cdot m^{-2}}$
It is also the intensity of the outgoing long-wave radiation at equilibrium.

The expression of the power emitted is given by Stefan-Boltzmann law:

$$P = \sigma \cdot e \cdot A \cdot T^4$$

Therefore,

$$T = (\frac{P}{\sigma \cdot e \cdot A})^{\frac{1}{4}}$$

$$P = I \cdot A$$

Considering the Earth as a black body: $e = 1$;

$$T = (\frac{250}{5.68 \times 10^{-8}})^{\frac{1}{4}} = \mathbf{258\ K = -\ 15°C}$$

Question 10:

a. Water has a specific **heat capacity** of

$$c_{\text{water}} = 4.18\ \text{J} \cdot \text{g}^{-1} \cdot \text{K}^{-1} = 4180\ \text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$$

it means that it requires 4.186 J of energy or 1 **calorie** to heat one gram of water by one degree.

The energy required to raise the temperature of a $h = 50$ m depth lake by $\Delta T = 1\ \text{K} = 1°\text{C}$ is:

$$E = m \cdot c_{\text{water}} \cdot \Delta T$$

$$E = \rho \cdot h \cdot S \cdot c_{\text{water}} \cdot \Delta T$$

ρ is the density of water:

$$\rho = 1\ \text{kg} \cdot \text{L}^{-1} = 1000\ \text{kg} \cdot \text{m}^{-3}$$

The intensity of the radiation is $I = 340\ \text{W} \cdot \text{m}^{-2}$; let's designate Δt the time required.

The energy that strikes the surface S of the lake during this time is: $E = I \cdot S \cdot \Delta t$.

Therefore:

$$\rho \cdot h \cdot S \cdot c_{\text{water}} \cdot \Delta T = I \cdot S \cdot \Delta t$$

$$\Delta t(\text{s}) = \frac{\rho(\text{kg} \cdot \text{m}^{-3}) \cdot h(\text{m}) \cdot c_{\text{water}}(\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}) \cdot \Delta T(\text{K})}{I(\text{W} \cdot \text{m}^{-2})}$$

$$\Delta t = \frac{1000 \times 50 \times 4180 \times 1}{340} = 614706\ \text{s} = \mathbf{171\ h}$$

b. If we take an average depth of 3000 m for the water on the surface of the earth and if we consider that 70% of the earth is covered with water and that the surface of the Earth is $S = 4\pi \cdot R^2$; the mass of water on the Earth is $m = \rho \cdot h \times 0.7 \times 4\pi \cdot R^2$ and the heat capacity of this body of water is:

$$C = m \cdot c_{\text{water}}$$

$$C = \rho \cdot h \times 0.7 \times 4\pi \cdot R^2 \cdot c_{\text{water}}$$

The radius of the Earth is: $R = 6370\ \text{km} = 6.37 \times 10^6\ \text{m}$

$$C = 1000 \times 3000 \times 0.7 \times 4\pi \cdot (6.37 \times 10^6)^2 \times 4180 = \mathbf{4 \times 10^{24}\ J \cdot K^{-1}}$$

c. We use the same reasoning as in question a.

$$\Delta t(\text{s}) = \frac{\rho(\text{kg} \cdot \text{m}^{-3}) \cdot h(\text{m}) \cdot c_{\text{water}}(\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}) \cdot \Delta T(\text{K})}{I(\text{W} \cdot \text{m}^{-2})}$$

$$\Delta t(\text{s}) = \frac{1000 \times 3000 \times 4180 \times 1}{340} = 4 \times 10^7\text{s} = 427\ \text{days} = \mathbf{1.2\ year}$$

Question 11:

In the simple model without considering the atmosphere, le power emitted by the planet is equal to the power that it absorbs.

Let's designate α the albedo of the planet.

We consider that the Solar power output is: $P = 3.9 \times 10^{26}$ W ;

The intensity at a distance d is:

$$I = \frac{P}{4\pi \cdot d^2}$$

The effective power that strikes the planet of radius R is:

$$I \cdot \pi \cdot R^2 = \frac{P}{4 \cdot d^2} \cdot R^2$$

The power that is absorbed is:

$$\frac{P}{4 \cdot d^2} \cdot R^2 \cdot (1 - \alpha)$$

The power that is emitted is given by Stefan-Boltzmann law:

$$P = e \cdot \sigma \cdot A \cdot T^4$$

$$\sigma = 5.68 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}$$

If we assume the planet is a black body: $e = 1$, the planet receives as much as it emits at equilibrium:

$$\sigma \cdot 4\pi \cdot R^2 \cdot T^4 = \frac{P}{4 \cdot d^2} \cdot R^2 \cdot (1 - \alpha)$$

$$T = \left(\frac{P \cdot (1 - \alpha)}{16\pi \cdot d^2 \cdot \sigma}\right)^{\frac{1}{4}}$$

Venus: the distance to the Sun is taken to be $d = 1.08 \times 10^{11}$ m and the albedo: $\alpha = 0.59$

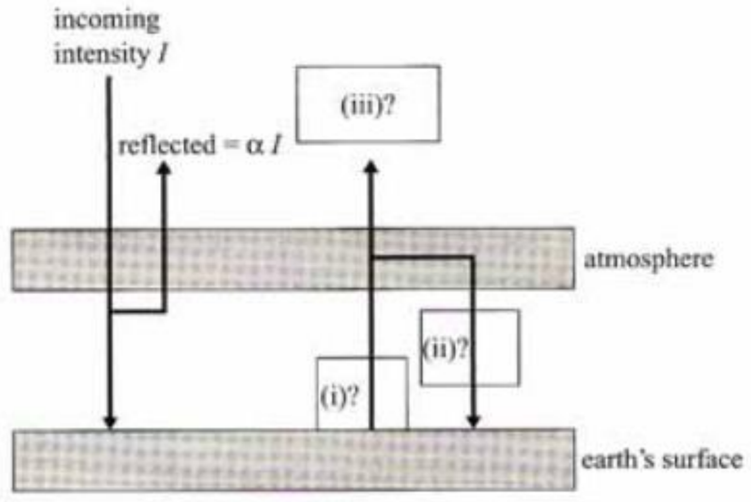
$$T_{\text{Venus}} = \left(\frac{3.9 \times 10^{26} \cdot (1 - 0.59)}{16\pi \times (1.08 \times 10^{11})^2 \times 5.68 \times 10^{-8}}\right)^{\frac{1}{4}} = \left(\frac{3.9 \times 0.41 \times 10^{12}}{16\pi \times (1.08)^2 \times 5.68}\right)^{\frac{1}{4}} = \mathbf{263\ K}$$

The actual temperature of Venus is: 740 K ; Venus has effectively a 90 times thicker atmosphere than the Earth.

Mars: the distance to the Sun is taken to be $d = 2.28 \times 10^{11}$ m and the albedo: $\alpha = 0.15$

$$T_{\text{Mars}} = \left(\frac{3.9 \times 10^{26} \cdot (1 - 0.15)}{16\pi \times (2.28 \times 10^{11})^2 \times 5.68 \times 10^{-8}}\right)^{\frac{1}{4}} = \left(\frac{3.9 \times 0.85 \times 10^{12}}{16\pi \times (2.28)^2 \times 5.68}\right)^{\frac{1}{4}} = \mathbf{217\ K}$$

The actual temperature of Mars is: 213 K ; Mars has no atmosphere (its atmosphere is 100 times thinner than the Earth).
Question 12:



(I): intensity radiated by the Earth as a hot body at its surface:

$$I_{\text{Earth}} = \frac{P}{4\pi \cdot R^2} = \frac{\sigma \cdot 4\pi \cdot R^2 \cdot T^4}{4\pi \cdot R^2} = \sigma \cdot T^4$$

(II): intensity that goes back to the Earth: $(1-t) \cdot I_{\text{Earth}}$

(III): intensity that leaves the earth: $t \cdot I_{\text{Earth}}$

The energy balance equation is:

$$I_{\text{Sun}} \cdot (1-\alpha) + (1-t) \cdot I_{\text{Earth}} = I_{\text{Earth}}$$

$$I_{\text{Sun}} \cdot (1-\alpha) = t \cdot I_{\text{Earth}}$$

$$t = \frac{I_{\text{Sun}} \cdot (1-\alpha)}{I_{\text{Earth}}} = \frac{I_{\text{Sun}} \cdot (1-\alpha)}{\sigma \cdot T^4}$$

$$t = \frac{350 \cdot (1-0.3)}{5.68 \times 10^{-8} \times 288^4} = \mathbf{0.63}$$

Question 20:

a. The total amount of reflected energy is : 27% : 18% (from the atmosphere) and 9% (from the surface); therefore the albedo is **0.27**.
b. The planet's temperature remains constant as the energy flowing downward (100%) is equal to the energy flowing upward (18% + 9% + 73%).

Question 23:

The change of albedo is: 0.3*(0.3-0.4) +0.1*(0.7-0.6) =-0.03+0.01=-0.02

As a change of albedo by - 0.01 leads to a 1°C increase of the temperature; the expected change in temperature due to the melting ice is a **2°C or 2K** increase.

Question 29:

a. Knowing the specific latent heat of fusion: 330 kJ.kg^{-1}; the amount of energy required to melt a 10^5 kg iceberg is: $\mathbf{3.3 \times 10^{10} J}$.
b. If an iceberg melts down, there will be no increase of the sea level; the ice displaces its own weight in water when it floats (Archimedes' principle), and when it melts, it fills the space that it previously displaced.

Question 30:

The volume of the Mediterranean Sea is: $2.5 \times 10^6 \times 1.5 = 3.75 \times 10^6$ km^3.
The coefficient of volume expansion of water is defined by the fractional change in volume per unit temperature change: 2×10^{-4} K^{-1}.
So, the change of volume of water is: $(2 \times 10^{-4} \times 3) \times 3.75 \times 10^6 = 2250$ km^3
The area doesn't change so the change of depth is: $\frac{2250}{2.5\times10^6} = 0.0009 \text{ km} = \mathbf{0.9\ m}$.

Question 31:

When icebergs (or any floating ice) melt, they're already displacing their mass in the water they're floating in. So, the melting of icebergs doesn't contribute to a net increase in water volume. However, if ice that's not already in the water (like ice sheets on land) melts, that does contribute to sea-level rise. Only land ice above sea level will contribute to sea level rise.

The density of ice is about 917 kilograms per cubic meter, and the density of seawater is about 1000 kilograms per cubic meter.

The water from the melting West Antarctic Ice Sheet would spread out and raise the level of all the seas and oceans around the world uniformly.

The water occupies 70% of the surface of the Earth which is: 510 million km² ($4\times \pi \times R^2$; $R = 6370$ km).

The volume of water is given by:

$$m_{water} = m_{ice}$$

$$V_{water} \times \rho_{water} = V_{ice} \times \rho_{ice}$$

$$V_{water} = S \times h$$

$$\mathrm{V}_{ice} = S \times h \times \frac{\rho_{\mathrm{water}}}{\rho_{\mathrm{ice}}}$$

$$\mathrm{V}_{ice} = 0.7 \times 510 \times 10^{6} \times 6 \times 10^{-3} \times \frac{1000}{917} = \mathbf{2.3 \times 10^{6}\ km^{3}}$$