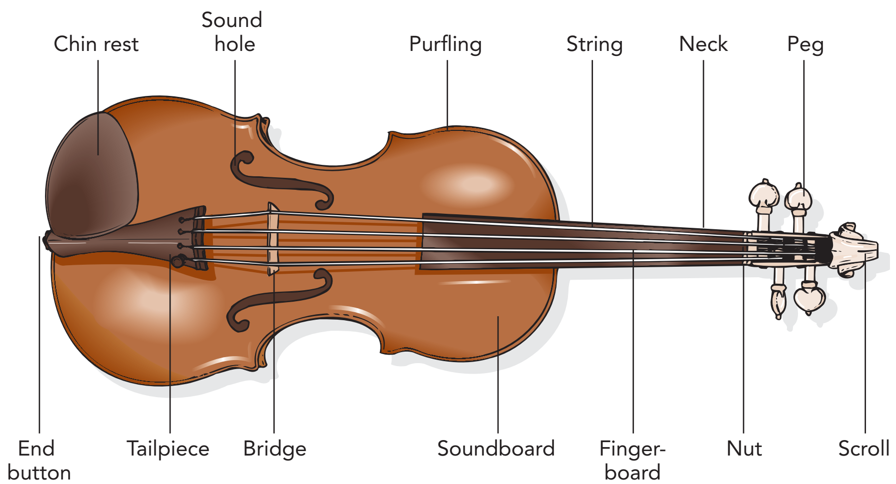
SECTION 9.2 Musical Instruments



Music is an important part of human expression. Although what qualifies as music is a matter of taste, it always involves sound and often involves instruments. In this section, we'll examine sound, music, and several instruments: violins, pipe organs, and drums. As examples of the three most common types of instruments—strings, winds, and percussion—this trio will help us to understand many other instruments as well.

Questions to Think About: Why are the low-pitch strings on a violin thicker than the high-pitch strings? How does pressing a violin's string against the fingerboard change its pitch? Why does a violin sound different when it's plucked rather than bowed? What purpose does the violin's body serve? What is vibrating inside a pipe organ? Why are some organ pipes longer than others? Why do most drums sound toneless, providing more rhythm than pitch?

Experiments to Do: Find a violin or guitar, or stretch a strong string between two rigid supports. Even a rubber band will do in a pinch. Pluck the string with your finger and listen to the tone it makes. The string vibrates back and forth at a particular frequency or pitch even as its amplitude of motion gradually decreases. What kind of oscillator has that behavior?

Change the string's frequency of vibration by changing its tension or length. What happens when you shorten the string or prevent part of it from moving? What happens if you increase its tension by pulling it tighter? You can also increase the string's mass by wrapping it with tape. How does that affect its pitch?

You can imitate a pipe organ by blowing gently across the mouth of a bottle or soda straw. If done properly, you'll get the air inside the bottle vibrating up and down rhythmically and you'll hear a tone. What happens to its pitch when you add water to the bottle or pinch off the straw at various points? Why does this tone sound different from that of the string, even when the two have the same pitch?

Finally, a table or plate will act like a drum when you tap it. Compare the sound it makes with those of the previous two "instruments." Does it have a pitch? What distinguishes the sounds of different tables or plates?

Sound and Music

To understand how instruments work, we need to know a bit more about sound and music. In air, **sound** consists of density waves, patterns of compressions and rarefactions that travel outward rapidly from their source. When a sound passes by, the air pressure in your ear fluctuates up and down about normal atmospheric pressure. Even when these fluctuations have amplitudes less than a millionth of atmospheric pressure, you hear them as sound.

When the fluctuations are repetitive, you hear a *tone* with a pitch equal to the fluctuation's frequency. **Pitch** is the frequency of a sound. A bass singer's pitch range extends from 80 to 300 Hz, while that of a soprano singer extends from 300 to 1100 Hz. Musical instruments can produce tones over a much wider range of pitches, but we can hear only those between about 30 and 20,000 Hz, and that range narrows as we get older.

Most music is constructed around *intervals,* the frequency ratio between two different tones. This ratio is found by dividing one tone's frequency by that of the other. Our hearing is particularly sensitive to intervals, with pairs of tones at equal intervals sounding quite similar to one another. For example, a pair of tones at 440 and 660 Hz sounds similar to a pair at 330 and 495 Hz because they both have the interval 3/2.

The interval 3/2 is pleasing to most ears and is common in Western music, where it's called a *fifth*. A fifth is the interval between the two "twinkles" at the beginning of "Twinkle, Twinkle, Little Star." If your ear is good, you can start with any tone for the first twinkle and will easily find the second tone, located at 3/2 the frequency of the first. Your ear hears that factor of 3/2 between the two frequencies.

The most important interval in virtually all music is 2/1, or an *octave*. Tones that differ by a factor of 2 in frequency sound so similar to our ears that we often think of them as being the same. When men and women sing together "in unison," they often sing an octave or two apart, and the differences in the tones, always factors of 2 or 4 in frequency, are only barely noticeable.

The octave is so important that it structures the entire range of audible pitches. Most of the subtle interplay of tones in music occurs in intervals of less than an octave, less than a factor of 2 in frequency. Thus most traditions build their music around the intervals that lie within a single octave, such as 5/4 and 3/2. They pick a particular standard pitch and then assign notes at specific intervals from this standard pitch. This arrangement repeats at octaves above and below the standard pitch to create a complete scale of notes. (For a history of scales, see 4.)

4 In addition to his contributions to mathematics, geometry, and astronomy, the Greek mathematician Pythagoras (ca. 580–500 BC) was perhaps the first person to use mathematics to relate intervals, pitches, and the lengths of vibrating strings. He and his followers laid the groundwork for the scale used in most Western music.

The scale used in Western music is constructed around a note called A_4, which has a standard pitch of 440 Hz. At intervals of 9/8, 5/4, 4/3, 3/2, 5/3, and 15/8 above A_4 lie the six notes B_4, $C^{\#}_5$, D_5, E_5, $F^{\#}_5$, and $G^{\#}_5$. Similar collections of six notes are built above A_5 (880 Hz), which has a frequency twice that of A_4, and above A_3 (220 Hz), which has a frequency half that of A_4. In fact, this pattern repeats above A_1 (55 Hz) through A_8 (7040 Hz).

Actually, Western music is built around 12 notes and 11 intervals that lie within a single octave. Five more intervals account for five additional notes, B^{b}_4, C_5, $D^{\#}_5$, F_5, and G_5. It's also not quite true that every note is based exclusively on its interval from A_4. While A_4 remains at 440 Hz, the pitches of the other 11 notes have been modified slightly so that they're at interesting and pleasing intervals from one another as well as from A_4. This adjustment of the pitches led to the *well-tempered scale* that has been the basis for Western music for the last several centuries.

Check Your Understanding #1: **A Night at the Opera**

A typical singing voice can cover a range of about two octaves—for example, from C_4 to C_6. How broad is this range of frequencies?

Answer: There is a factor of 4 in frequency between the lowest and the highest notes that the typical voice can sing.

Why: Since notes separated by an octave are separated by a factor of 2 in frequency, notes separated by two octaves are separated by a factor of 4.

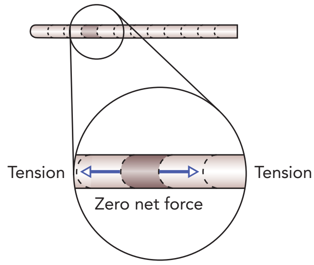


Fig. 9.2.1 A taut violin string can be viewed as being composed of many individual pieces. When the string is straight, the two forces exerted on a given piece by its neighbors cancel perfectly.

A Violin's Vibrating String

The tones produced by a violin begin as vibrations in its strings. On their own, these strings are limp and shapeless so they rely on the violin's rigid body and neck for structure. The violin subjects its strings to **tension**, outward forces that act to stretch it, and this tension gives each string an equilibrium shape—a straight line.

To see that a straight violin string is in equilibrium, think of it as being composed of many individual pieces that are connected together in a chain (Fig. 9.2.1). Tension exerts a pair of outward forces on each piece of the string; its neighboring pieces are pulling that piece toward them. Since the string's tension is uniform, these two outward forces sum to zero; they have equal magnitudes but point in opposite directions. With zero net force on each of its pieces, the straight string is in equilibrium.

When the string is curved, however, the pairs of outward forces no longer sum to zero (Fig. 9.2.2). Although those outward forces still have equal magnitudes, they now point in slightly different directions. As a result, each piece experiences a small net force.

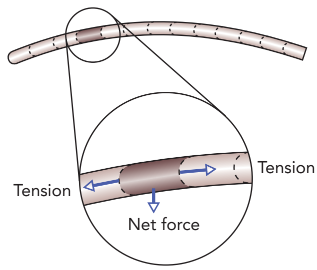


Fig. 9.2.2 When a violin string is curved, the two forces exerted on a given piece by its neighbors don't point in exactly opposite directions and don't balance one another. The piece experiences a net force.

The net forces on its pieces are restoring forces because they act to straighten the string. If you distort the string and release it, these restoring forces will cause the string to vibrate about its straight equilibrium shape in a natural resonance. The string's restoring forces are special; the more you curve the string, the stronger the restoring forces on its pieces become. In fact, the restoring forces are **springlike forces**—they increase in proportion to the string's distortion—so the string is a form of harmonic oscillator!

Actually, the string is much more complicated than a pendulum or a balance ring. It can bend and vibrate in many distinct **modes**, or basic patterns of distortion, each with its own period of vibration. Nonetheless, the string retains the most important feature of a harmonic oscillator: the period of each vibrational mode is independent of its amplitude. Thus a violin string's pitch doesn't depend on how hard it's vibrating. Think how tricky it would be to play a violin if its pitch depended on its volume!

A violin string has one simplest vibration—its **fundamental vibrational mode**. In this mode, the entire string arcs alternately one way and then the other (Fig. 9.2.3). Its kinetic energy peaks as it rushes through its straight equilibrium shape, and its potential energy (elastic potential energy in the string) peaks as it stops to turn around. The string's midpoint travels the farthest (the **vibrational antinode**), while its ends remain fixed (the **vibrational nodes**). At each moment, its shape is the gradual curve of the trigonometric sine function.

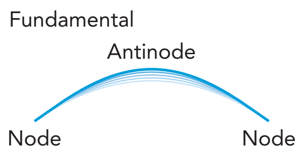


Fig. 9.2.3 A string vibrating between two fixed points in its fundamental vibrational mode. The whole string moves together, traveling up and down as a single harmonic oscillator.

In this fundamental mode, the violin string behaves as a single harmonic oscillator. As with any harmonic oscillator, its vibrational period depends only on the stiffness of its restoring forces and on its inertia. Either stiffening the violin string or reducing its mass will quicken its fundamental vibration and increase its fundamental pitch.

A violin has four strings, each with its own stiffness and mass, and therefore with its own fundamental pitch. In a tuned violin, the notes produced by these strings are G_3 (196 Hz),

D_4 (294 Hz), A_4 (440 Hz), and E_5 (660 Hz). The G_3 string, which vibrates rather slowly, is the most massive. It's usually made of gut, wrapped in a coil of heavy metal wire. The E_5 string, on the other hand, must vibrate quite rapidly and so needs to have a low mass. It's usually a thin steel wire.

You tune a violin by adjusting the tension in its strings using the pegs in its neck and tension adjusters on the tailpiece. Tightening the string stiffens it by increasing both the outward forces on its pieces and the net forces they experience during a distortion. Since temperature and time can alter a string's tension, you should always tune your violin just before a concert.

A string's fundamental pitch also depends on its length. Shortening the string both stiffens it and reduces its mass, so its pitch increases. That stiffening occurs because a shorter string curves more sharply when it's displaced from equilibrium and therefore subjects its pieces to larger net forces. This dependence on length allows you to raise a string's pitch by pressing it against the fingerboard in the violin's neck and effectively shortening it. Part of a violinist's skill involves knowing exactly where on the string to press it against the fingerboard to produce a particular note.

If the arc of a string vibrating in its fundamental mode reminds you of a wave, that's because it is one. It's a **mechanical wave**, the natural motions of an extended object about its stable equilibrium shape or situation. An *extended* object is one like a string, stick, or lake surface that has many parts that move with limited independence. Since its parts influence one another, an extended object with a stable equilibrium exhibits fascinating natural motions that involve many parts moving at once; it exhibits mechanical waves.

With its innumerable linked pieces and its stable equilibrium shape, the violin string exhibits such waves. The string's fundamental mode is a particularly simple wave, a **standing wave**, which is a wave with fixed nodes and antinodes. A standing wave's basic shape doesn't change with time; it merely scales up and down rhythmically at a particular frequency and amplitude (its peak extent of motion). Most important, the standing wave doesn't travel along the string.

Although this wave extends along the string, its associated oscillation is *perpendicular* to the string and therefore *perpendicular* to the wave itself. A wave in which the underlying oscillation is perpendicular to the wave itself is called a **transverse wave** (Fig. 9.2.4). Waves on strings, drums, and the surface of water are all transverse waves.

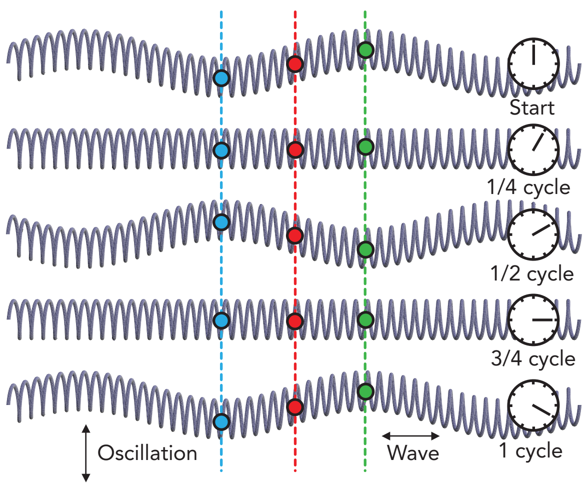


Fig. 9.2.4 In a transverse wave, the underlying oscillation is perpendicular to the wave itself. In this case, a spring is oscillating vertically but forming a horizontal wave. With its fixed nodes and antinodes, this transverse wave is also a standing wave.

Check Your Understanding #2: Feeling Tense?

A common way to determine the tension in a cord is to pluck it and listen for how fast it vibrates. Why does this technique measure tension?

Answer: The frequency of a string's fundamental vibrational mode increases with its tension.

Why: Any cord that is drawn taut from its ends will exhibit natural resonances like those in a violin string. The tauter the string, the higher will be the frequencies of those resonances.

The Violin String's Harmonics

The fundamental vibrational mode isn't the only way in which a violin string can vibrate. The string also has **higher-order vibrational modes** in which the string vibrates as a chain of shorter strings arcing in alternate directions (Fig. 9.2.5). Each of these higher-order vibrational modes is another standing wave, with a fixed shape that scales up and down rhythmically at its own frequency and amplitude.

For example, the string can vibrate as two half-strings arcing in opposite directions and separated by a motionless vibrational node. In this mode, the violin string not only vibrates as half-strings but has the pitch of half-strings as well. Remarkably, that half-string pitch is exactly twice the whole-string (fundamental) pitch! In general, a string's vibrational frequency is inversely proportional to its length, so halving its length doubles its frequency. Frequencies that are integer multiples of the fundamental pitch are called **harmonics**, so this half-string vibration occurs at the second harmonic pitch and is called the *second harmonic mode*.

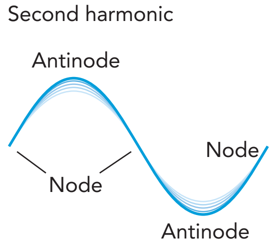
A violin string can also vibrate as three third-strings, with a frequency that's three times the fundamental. The interval between this third harmonic pitch and the fundamental pitch is an octave and a fifth (2/1 times 3/2). Overall, the fundamental and its second and third harmonics sound very pleasant together.

While the violin string can vibrate in even higher harmonics, what's more important is that the string often vibrates in more than one mode at the same time. For example, a violin string vibrating in its fundamental mode can also vibrate in its second harmonic and emit two tones at once.

Harmonics are important because bowing a violin excites many of its vibrational modes. The violin's sound is thus a rich mixture of the fundamental tone and the harmonics. Known as **timbre**, this mixture of tones is characteristic of a violin, which is why an instrument producing a different mixture doesn't sound like a violin.

When a violin string is vibrating in several modes at once, its shape and motion are complicated. The individual standing waves add on top of one another, a process known as **superposition**. Each vibrational mode has its own amplitude and therefore its own volume contribution to the string's timbre.

While these individual waves coexist beautifully on the string, with virtually no effect on one another, the string's overall distorted shape is now the superposition of the individual wave shapes. Not only is that overall shape quite complicated, it also actually changes substantially with time. That's because the different harmonic waves vibrate at different frequencies, and their superposition changes as they change. The string's overall wave is not a standing wave, and its features can even move along the string!



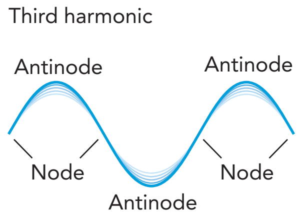


Fig. 9.2.5 A string vibrating between two fixed points in its second and third harmonic modes. The string vibrates as two or three segments, completing cycles at two or three times the fundamental frequency, respectively.

Check Your Understanding #3: Swinging High and Low Together

When two people swing a long jump rope, they can make it swing as a single arc or as two half-ropes arcing in opposite directions. To make the rope swing as two half-ropes, it must be turned faster or with less tension. Why?

Answer: A jump rope is essentially a vibrating string. The two-half-rope pattern is the second harmonic mode, with a vibrational frequency twice that of the normal fundamental arc.

Why: Although it swings around in a circle, the jump rope is actually vibrating up and down at the same time it's vibrating forward and back. Together, these two vibrations create the circular motion. To make the rope vibrate in its second harmonic mode (as two half-ropes) without changing its tension, it must be swung twice as fast as normal.

Bowing and Plucking the Violin String

You play a violin by drawing a bow across its strings. The bow consists of horsehair, pulled taut by a wooden stick and coated with rosin, a sticky substance based on pine sap. This coated horsehair exerts frictional forces on the strings as it moves across them. Most important, however, is that it exerts much larger static frictional forces than sliding ones.

As the sticky bow hairs rub across a string, they grab the string and push it forward with static friction. Eventually the string's restoring force overpowers static friction, and the string suddenly starts sliding backward across the hairs. Because the hairs exert little sliding friction, the string completes half a vibrational cycle with ease. As it stops to reverse direction, however, the hairs grab the string again and begin pushing it forward. This process repeats over and over.

Fig. 9.2.6 Resonant energy transfer makes it possible for sound to shatter a crystal wineglass. When the sound pushes on the glass rhythmically, the sound slowly transfers energy to the glass, until it finally shatters. Because the sound must be extremely loud and at exactly the resonant frequency of the glass, only the most extraordinary opera singers can break a crystal wineglass.

Each time the bow pushes the string forward, it does work on the string and adds energy to the string's vibrational modes. This process is an example of **resonant energy transfer**, in which a modest force doing work in synchrony with a natural resonance can transfer a large amount of energy to that resonance. Just as gentle, carefully timed pushes can get a child swinging high on a playground swing, so too can gentle, carefully timed pushes from a bow get a string vibrating vigorously on a violin. Similar rhythmic pushes can cause other objects to vibrate strongly, notably a crystal wineglass (Fig. 9.2.6) and the Tacoma Narrows Bridge near Seattle, Washington (Fig. 9.2.7). The wineglass's response to a certain tone is also an example of **sympathetic vibration**, the transfer of vibrational energy between two systems that share a common vibrational frequency.

The amount of energy the bow adds to each vibrational mode depends on where it crosses the violin string. When you bow the string at the usual position, you produce a strong fundamental vibration and a moderate amount of each harmonic. Bowing the string nearer its middle reduces the string's curvature, weakening its harmonic vibrations and giving it a more mellow sound. Bowing the string nearer its end increases the string's curvature, strengthening its harmonic vibrations and giving it a brighter sound.

The sound of a plucked violin string also depends on harmonic content and thus on where that string is plucked. However, this sound is quite different from that of a bowed string. The difference lies in the sound's *envelope,* the way the sound evolves with time. This envelope can be viewed as having three time periods: an initial attack, an intermediate sustain, and a final decay. The envelope of a plucked string is an abrupt attack followed immediately by a gradual decay. In contrast, the envelope of a bowed string is a gradual attack, a steady sustain, and then a gradual decay. We learn to recognize individual instruments not only by their harmonic content but also by their sound envelopes.

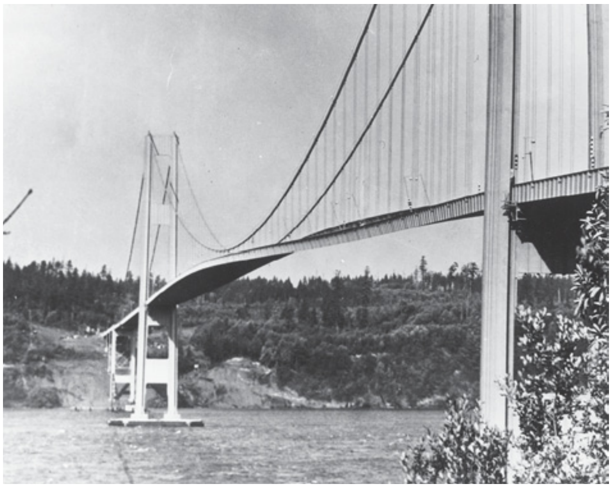


Fig. 9.2.7 The Tacoma Narrows Bridge collapsed in November 1940, as the result of resonant energy transfer between the wind and the bridge surface. Shortly after its construction, the automobile bridge began to exhibit an unusual natural resonance in which its surface twisted slowly back and forth so that one lane rose as the other fell. During a storm, the wind slowly added energy to this resonance until the bridge ripped itself apart.

Check Your Understanding #4: What's the Buzz?

Sometimes a tone from an instrument or sound system will cause some object in the room to begin vibrating loudly. Why does this happen?

Answer: The object has a natural resonance at the tone's frequency, and sympathetic vibration is transferring energy to the object.

Why: Energy moves easily between two objects that vibrate at the same frequency. A note played on one instrument will cause the same note on another instrument to begin playing. Even everyday objects will exhibit sympathetic vibration when the right tone is present in the air.

An Organ Pipe's Vibrating Air

Like a violin, a pipe organ uses vibrations to create sound. However, its vibrations take place in the air itself. An organ pipe is essentially a hollow cylinder, open at each end and filled with air. Because that air is protected by the rigid walls of the pipe, its pressure can fluctuate up and down relative to atmospheric pressure and it can exhibit natural resonances.

In its fundamental vibrational mode, air moves alternately toward and away from the pipe's center (Fig. 9.2.8), like two blocks on a spring. As air moves toward the pipe's center, the density there rises and a pressure imbalance develops. Since the pressure at the pipe's center is higher than at its ends, air accelerates *away* from the center. The air eventually stops moving inward and begins to move outward. As air moves away from the pipe's center, the density there drops and a reversed pressure imbalance occurs. Since the pressure at the pipe's center is lower than at its ends, air now accelerates *toward* the center. It eventually stops moving outward and begins to move inward, and the cycle repeats. The air's kinetic energy peaks each time it rushes through that equilibrium and its potential energy (pressure potential energy in the air) peaks each time it stops to turn around.

This air is vibrating about a stable equilibrium of uniform atmospheric density and pressure, and it is clearly experiencing restoring forces. It should come as no surprise that

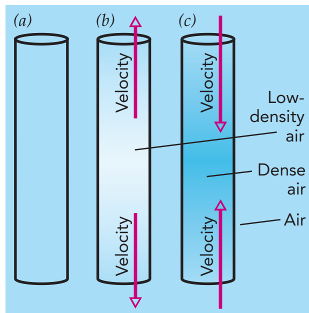


Fig. 9.2.8 In a pipe that's open at both ends (*a*), the air vibrates in and out about the middle of the pipe. (*b*) For half a cycle the air moves outward and creates a low-pressure region in the middle, and (*c*) for half a cycle the air moves inward and creates a high-pressure region there.

those restoring forces are springlike and that the air column is yet another harmonic oscillator. As such, its vibrational frequency depends only on the stiffness of its restoring forces and on its inertia. Either stiffening the air column or reducing its mass will quicken its vibration and increase its pitch.

These characteristics depend on the length of the organ pipe. A shorter pipe not only holds less air mass than a longer pipe, it also offers stiffer opposition to any movements of air in and out of that pipe. With less room in the shorter pipe, the pressure inside it rises and falls more abruptly, leading to stiffer restoring forces on the moving air. Together, these effects make the air in a shorter pipe vibrate faster than the air in a longer pipe. In general, an organ pipe's vibrational frequency is inversely proportional to its length.

Unfortunately, the mass of vibrating air in a pipe also increases with the air's average density, so even a modest change in temperature or weather will alter the pipe's pitch. Fortunately, all the pipes shift together so that an organ continues to sound in tune. Nonetheless, this shift may be noticeable when the organ is part of an orchestra.

As you may suspect, the fundamental vibrational mode of air in the organ pipe is another standing wave. Air in the pipe is an extended object with a stable equilibrium, and the disturbance associated with its fundamental vibrational mode has a basic shape that doesn't change with time; it merely scales up and down rhythmically.

However, the shape of the wave in the pipe's air now has to do with back-and-forth compressions and rarefaction, not with side-to-side displacements, as it did in the violin string. In fact, all the wave's associated oscillation is *along* the pipe and therefore *along* the wave itself. A wave in which the underlying oscillation is parallel to the wave itself is called a **longitudinal wave** (Fig. 9.2.9). Waves in the air, including those inside organ pipes and other wind instruments and those in the open air, are all longitudinal waves.

Check Your Understanding #5: A Pop Organ

If you blow across a soda bottle, it emits a tone. Why does adding water to the bottle raise the pitch of that tone?

Answer: The water shortens the column of moving air inside the bottle and increases the frequency of its fundamental vibrational mode.

Why: A water bottle is essentially a pipe that is open at only one end. It has a fundamental vibrational mode with a frequency that is half that of an open pipe of equal length. As you add water to the bottle, you shorten the effective length of the pipe and raise its pitch.

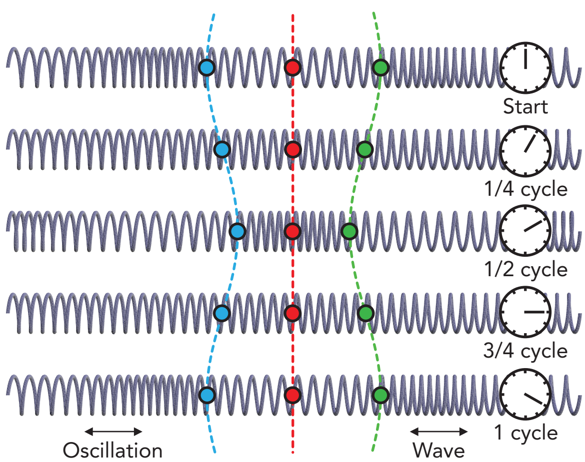


Fig. 9.2.9 In a longitudinal wave, the underlying oscillation is parallel to the wave itself. In this case, a spring is oscillating horizontally, in the same direction as the wave. With its fixed nodes and antinodes, this longitudinal wave is also a standing wave.

Playing an Organ Pipe

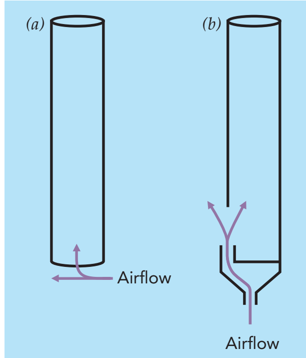


Fig. 9.2.10 (*a*) Air blown across the bottom of an open pipe will follow any other air that's moving into the pipe. If the air in the pipe is vibrating, this effect will add energy to that vibration. (*b*) The lower opening in an organ pipe is cut in its side for practical reasons.

The organ uses resonant energy transfer to make the air in a pipe vibrate. It starts this transfer by blowing air across the pipe's lower opening (Fig. 9.2.10*a*), although for practical reasons that lower opening is usually found on the pipe's side (Fig. 9.2.10*b*). As the air flows across the opening, it's easily deflected to one side or the other and tends to follow any air that's already moving into or out of the pipe. If the air inside the pipe is vibrating, the new air will follow it in perfect synchrony and strengthen the vibration.

This following process is so effective at enhancing vibrations that it can even initiate a vibration from the random noise that's always present in a pipe. That's how the sound starts when the organ's pump first blows air across the pipe. Once the vibration has started, it grows quickly in amplitude. That amplitude increases until energy leaves the pipe as sound and heat as quickly as it arrives via compressed air. The more air the organ blows across the pipe each second, the more power it delivers to the pipe and the louder the vibration.

Like a violin string, an organ pipe can support more than one mode of vibration. In its fundamental vibrational mode, the pipe's entire column of air vibrates together. In the higher-order vibrational modes, this air column vibrates as a chain of shorter air columns moving in alternate directions. If the pipe has a constant width, these vibrations occur at harmonics of the fundamental. When the air column vibrates as two half-columns, its pitch is exactly twice that of the fundamental mode. When it vibrates as three third-columns, its pitch is exactly three times that of the fundamental. And so on.

Also, the air column inside a pipe can vibrate in more than one mode simultaneously. As with a violin string, the standing waves superpose and the fundamental and harmonic tones are produced together. The shape of the organ pipe and the place where air is blown across it determine the pipe's harmonic content and thus its timbre. Different pipes can imitate different instruments. To sound like a flute, the pipe emits mostly the fundamental tone and keep the harmonics fairly quiet. To sound like a clarinet, its harmonics must be much louder. An organ pipe's volume always builds slowly during the attack, so it can't pretend to be a plucked string. However, a clever designer can make the organ imitate a surprising range of instruments.

Check Your Understanding #6: **An Across Blow**

To make the air in a soda bottle vibrate, you must blow *across* the bottle's mouth. Why doesn't blowing *into* its mouth work?

Answer: By blowing across the mouth, you let air that is already vibrating in the bottle redirect your breath so that it enhances the vibration. Blowing into the bottle's mouth merely compresses the air inside the bottle.

Why: Like the bow of a violin moving across its strings, your breath moving across the bottle's mouth enhances the air's vibration via resonant energy transfer. The spontaneous redirection of your breath when you blow across the bottle's mouth leads to rhythmic pushes that are perfectly synchronized with the air's vibration.

A Drum's Vibrating Surface

After examining violin strings and organ pipes, you might think that drums have no new physics concepts to show us. But while a drumhead is yet another extended object with a stable equilibrium and springlike restoring forces, its overtone vibrations have an important difference: they *aren't* harmonics.

Violin strings and organ pipes are effectively one-dimensional or linelike objects, dividing easily into half-objects or third-objects that then vibrate at second or third harmonic pitches. Together with the many other one-dimensional instruments in an orchestra or band, they blend seamlessly when they're playing the same fundamental pitch because they share the same harmonics.

However, because a drumhead is effectively two-dimensional or surfacelike, it doesn't divide easily into pieces that resemble the entire drumhead. As a result, the pitches of its overtone vibrations have no simple relationship to its fundamental pitch. A timpani stands out relative to other instruments in part because of the unique overtone pitches.

Figure 9.2.11 illustrates the fundamental (Fig. 9.2.11*a*) and five lowest-pitched overtone (Fig. 9.2.11*b*–*f*) vibrational modes for a drumhead. Each vibrational mode is a standing wave but with vibrational nodes that are curves or lines rather than points. The fundamental mode (Fig. 9.2.11*a*) has only one node on its outer edge, while the overtone modes have additional nodes within the surface. In each vibrational mode, these nodes remain motionless as the rest of the surface vibrates up and down, its peaks and valleys interchanging alternately. The pitches of the overtone vibrations are indicated relative to the pitch of the fundamental vibration. (For a historical note on the understanding of surface modes, see 5).

5 In 1809, the French Academy of Sciences announced a competition to explain the intricate patterns observed on vibrating surface plates. The only respondent was French mathematician Sophie Germain (1776–1831). As a woman, Germain had been barred from formal education in mathematics and had struggled to learn the subject from books and via correspondence with leading mathematicians, which she conducted under the pseudonym Antoine-August Le Blanc. It took her three tries, but in 1816 she was awarded the prize. Because she was a woman, however, she did not attend the ceremony. Her analysis of surface vibrations, though imperfect, was a visionary effort, made all the more extraordinary by her circumstances. Although her mentor, Carl Friedrich Gauss, managed to convince the University of Göttingen to award her an honorary degree, she died of breast cancer before she could receive it.

Because striking a drumhead causes it to vibrate in several modes at once, the drum emits several pitches simultaneously. The amplitude of each mode, and consequently its volume, depends not only on *how hard* you hit the drumhead but also on *where* you hit it. If you hit it at its center, it vibrates primarily in circular modes (Fig. 9.2.11*a,d*). If you hit it nearer its edge, it also vibrates in noncircular modes (Fig. 9.2.11*b,c,e,f*).

A timpani sounds most musical when it's struck off-center in such a way that the amplitude of its fundamental vibrational mode is nearly zero and its overtones, particularly Fig. 9.2.11*b*, dominate its sound. That's because the fundamental vibrational mode emits sound so efficiently that its vibrational energy dissipates before it can produce a discernible tone. Unless all you want is a loud thump, you must hit the timpani off-center so that its long-lived overtone vibrations receive most of the energy and emit most of the sound. The dominant pitch of a properly played timpani is that of its first overtone vibration, and it is tuned with that pitch in mind.

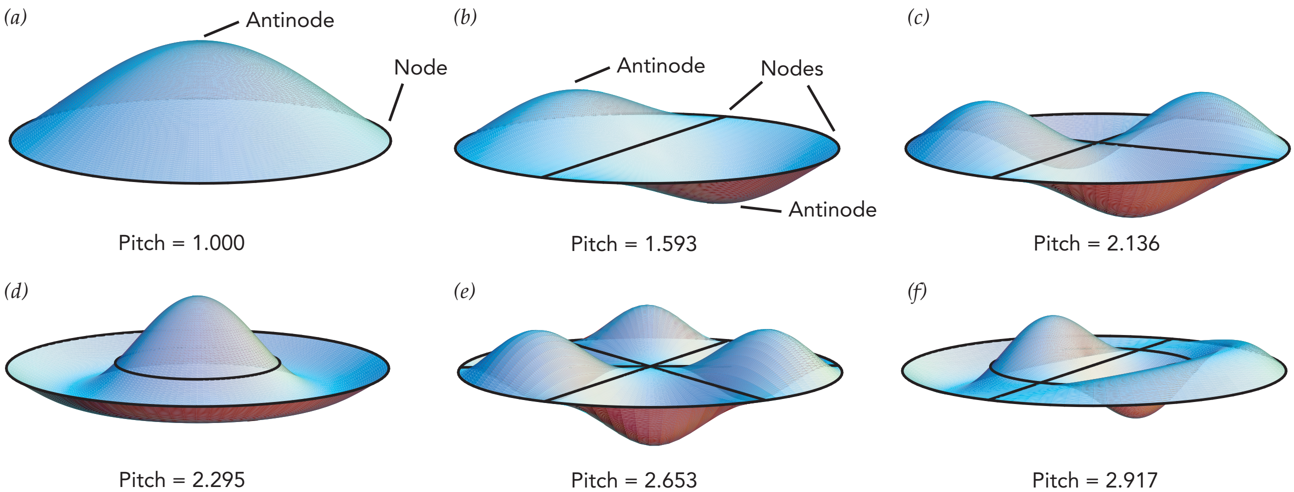


Fig. 9.2.11 The six lowest-pitch vibrational modes of a drumhead, including (*a*) the fundamental vibrational mode and (*b*–*f*) overtone modes. Pitches are shown relative to the fundamental pitch.

In truth, the pitches shown in Fig. 9.2.11 neglect the effects of air's inertia on the drumhead's vibrations. Since air adds inertia to the drumhead, it lowers the pitches of all the vibrational modes, some more than others. Because of air's influence on pitch, a drum must be tuned to accommodate changes in temperature and weather.

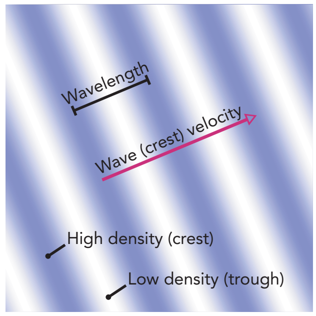


Fig. 9.2.12 A traveling sound wave in air consists of a washboard pattern of high-density (blue) and low-density (white) regions. The distance separating adjacent crests is the wavelength, and the speed and direction of crest motion are the wave velocity.

Check Your Understanding #7: **Space Kid One**

A trampoline is hazardous with several children on it because a child landing on one side of its surface can launch skyward a second child standing on the other side of the surface. How does a downward impact on one side of the trampoline produce a sudden rise of the other side?

Answer: The off-center impact causes the surface to vibrate in its noncircular overtone modes. The simplest such mode, Fig. 9.2.11*b*, has its two sides moving in alternate directions.

Why: The trampoline is essentially a drumhead, and the children are riding its vibrational modes. Off-center impacts can cause the surface to vibrate in its overtone modes, and these can toss the children in unexpected directions.

Sound in Air

All these vibrations would serve little purpose if we couldn't hear them, so it's time to look at how instruments produce sound. We'll start by looking at sound itself.

We noted at the beginning of this section that sound in air consists of density waves—patterns of compressions and rarefactions that travel outward rapidly from their source. While that observation was mysterious at the time, we can now understand those waves as vibrations in an extended object with a stable equilibrium. That extended object is air.

Neglecting gravity, air is in a stable equilibrium when its density is uniform. If we disturb it from equilibrium, the resulting pressure imbalances will provide springlike restoring forces. These forces, together with air's inertia, lead to rhythmic vibrations—the vibrations of harmonic oscillators. In open air, the most basic vibrations are waves that move steadily in a particular direction and are therefore called **traveling waves**. Like the standing waves inside an organ pipe, the traveling waves in open air are longitudinal—air vibrates along the same direction as the sound wave travels.

As it moves through the open air, a basic traveling sound wave consists of an alternating pattern of high-density regions we'll call **crests** and low-density regions we'll call **troughs** (Fig. 9.2.12). While those names will seem more appropriate when we examine water surface waves in the next section, it's customary to refer to the alternating highs and lows of any wave as crests and troughs, respectively. Whether a wave is standing or traveling, the shortest distance between two adjacent crests is known as the **wavelength**.

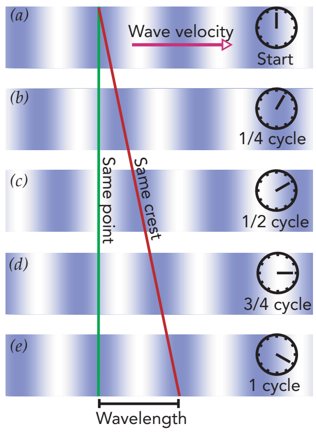


Fig. 9.2.13 A sound wave at five evenly spaced times (*a–e*) showing one complete cycle of oscillation. During that cycle, the pressure at a specific point in space goes from high to low to high (green line), and a specific crest moves 1 wavelength to the right (red line).

A standing wave's crests and troughs merely flip back and forth in place, crests becoming troughs and troughs becoming crests. However, a traveling wave's crests and troughs move steadily in a particular direction at a particular speed. The speed and direction of travel together constitute the traveling wave's **wave velocity**.

Figure 9.2.13 shows five snapshots of a simple sound wave that's heading toward the right. If we watch air's density at the same point in space (green line), it begins as a crest (*a*), decreases (*b*) to a trough (*c*), then rises (*d*) back to a crest (*e*) during one complete vibration cycle. However, if we follow the same crest (red line) over time, it travels one wavelength to the right during one complete vibration cycle (*a–e*). Since a crest moves one wavelength per vibration cycle and frequency is the number of vibration cycles per second, the speed at which the crest moves is equal to the wavelength times the frequency. That relationship can be written as a word equation:

$$\text{wave speed} = \text{wavelength} \cdot \text{frequency} \tag{9.2.1}$$

in symbols:

$$s = \lambda \nu,$$

and in everyday language:

Broad waves that vibrate quickly travel fast.

Remarkably enough, all sound waves travel at the same speed through air, regardless of wavelength or frequency. That's because as a sound wave's wavelength increases, its crests travel farther during one cycle of vibration but that cycle also takes longer. The two changes balance one another, so the crests travel at the same speed. Longer wavelengths lead to slower vibrations because broadening out the pressure variations in the sound wave weakens its restoring forces. With softer restoring forces and the same inertia (that is, its density), the air vibrates more slowly as the wavelength increases.

Equation 9.2.1 thus yields the same wave speed for any sound wave. Known as the **speed of sound** in air, it's about 331 m/s (1086 ft/s) under standard conditions at sea level (0 °C, 101, 325 Pa pressure). Although that's fast, there is still a noticeable delay between when a percussionist strikes the cymbals and when you hear them from across the concert hall. Fortunately, because the speed of sound doesn't depend on frequency, when the entire orchestra plays in unison you hear all its different pitches simultaneously.

This discussion of sound assumes that the instruments and listener maintain a constant separation, as they usually do at an orchestra concert. However, when a marching band steps quickly toward or away from the listener, something odd happens: the listener hears its music shifted up or down in pitch. Known as the **Doppler effect**, this frequency shift occurs because the listener encounters sound wave crests at a rate that's different from the rate at which those crests were created. If an instrument and the listener are approaching one another, the listener encounters the crests at an increased rate and the pitch increases. If the two are moving away from one another, the listener encounters the crests at a decreased rate and the pitch decreases. Fortunately, the Doppler effect is subtle at speeds that are small compared to the speed of sound, so you can listen to parades without their sounding flat or sharp.

Check Your Understanding #8: Sounds Faster

Helium in a toy balloon has the same stiffness as ordinary air, but its density and inertia are smaller. How does this difference affect the speed of sound in helium?

Answer: Sound travels faster in helium than it does in ordinary air.

Why: With its reduced density and inertia, helium vibrates faster than air when the two gases carry sound waves of equal wavelengths. Since the speed at which sound of a specific wavelength travels is proportional to the frequency of that sound, the wave speed in helium is greater than that in air.

Check Your Figures #1: Underwater Sound

Although water is about 800 times as dense as ordinary air, water is also about 15,000 times as stiff and its sound vibrations therefore have increased frequencies relative to those in air. When two sound waves have equal wavelengths, the wave in water has a frequency about 4.3 times greater than the wave in air. What is the speed of sound in water?

Answer: The speed of sound in water is about 1420 m/s (4700 ft/s).

Why: From Eq. 9.2.1, the wave speed in water must be 4.3 times the wave speed in air. Since the sound waves have a wave speed of about 331 m/s in air, they must have a speed of about 331 m/s times 4.3 or 1420 m/s in water.

Turning Vibrations into Sound

Anything that disturbs air's otherwise uniform density can produce traveling sound waves. Instruments emit sound by compressing and rarefying the nearby air in synch with their own vibrations. How they accomplish this task differs from instrument to instrument, so we'll have to look at them individually. As we'll see, some instruments find it easier to produce sound than others.

A drum produces sound when its vibrating drumhead alternately compresses and rarefies the nearby air. As portions of that drumhead rise and fall, they upset the air's uniform density and thereby produce sound waves. Whenever it can, though, air simply flows silently out of the drumhead's way, leading to smaller density fluctuations and less intense sound. For example, when the drumhead is experiencing one of the five overtone vibrations shown in Fig. 9.2.11, air flows away from each rising peak in the undulating surface and toward each falling valley. The overtone vibrations still manage to produce sound, but it's less intense and the vibrational energy in the drumhead transforms relatively inefficiently into sound energy.

Air's partial success in dodging the drumhead's overtone vibrations allows those overtones to complete many vibrational cycles before running out of vibrational energy. Their vibrations are therefore long-lived and have distinct pitches. In contrast, air has difficulty dodging the drumhead's fundamental vibrational mode, which alternately compresses and rarefies the air so effectively that it transfers all its vibrational energy to the air in just a few cycles. That's why the fundamental vibrational mode of a drumhead produces the intense and nearly pitchless "thump" sound that we associate with a drum.

If air can dodge a vibrating surface, it can certainly dodge a vibrating string. Little of a violin's sound comes directly from its vibrating strings; the air simply skirts around them. Instead, the violin creates sound with its top plate, or *belly* (Fig. 9.2.14). The strings transfer their vibrational motions to the belly and the belly pushes on the air to create sound.

Fig. 9.2.14 A violin's bridge transfers energy from its vibrating strings to its belly. The belly moves in and out, emitting sound. Some of this sound leaves the violin through the *S*-holes in its body.

Most of this vibrational energy flows into the belly through the violin's *bridge*, which holds the strings away from the violin's body (Fig. 9.2.15). Beneath the G_3-string side of the bridge is the *bass bar*, a long wooden strip that stiffens the belly. Beneath the E_5-string side of the bridge is the *sound post*, a shaft that extends from the violin's belly to its back.

As a bowed string vibrates across the violin's belly, it exerts a torque on the bridge about the sound post. The bridge rotates back and forth, causing the bass bar and belly to move in and out. The belly's motion produces most of the violin's sound. Some of this sound comes directly from the belly's outer surface, and the rest comes from its inner surface and must emerge through its *S*-shaped holes.

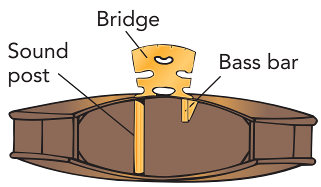


Fig. 9.2.15 The bridge is supported by the bass bar on one side and the sound post on the other. As the strings vibrate back and forth, the bridge experiences a torque that causes the belly of the violin to move in and out and emit sound.

An organ pipe doesn't have to produce sound because that sound already exists. In effect, the pipe's vibrating column of air is a standing sound wave that gradually leaks out of the pipe as a traveling one. Trapped sound is escaping from its container.

This conversion of a standing wave into a traveling wave isn't so remarkable because the two types of waves are closely related. The pipe's standing wave can be thought of as a reflected traveling wave, a traveling wave that's bouncing back and forth between the two ends of the pipe. Because of the reflections, the traveling wave is superposed with itself heading in the opposite direction, and the sum of two equal but oppositely directed traveling waves *is* a standing wave!

The fact that sound reflects from the open end of an organ pipe is rather surprising. If that end were closed, you'd probably expect a reflection. After all, sound echoes from cliffs and other rigid surfaces. But sound partially reflects from a surprising range of other transitions, including the transition from inside a pipe to outside it. If you don't believe that, clap your hands inside a long pipe and listen for the decaying echoes.

The reflections at the organ pipe's open ends aren't perfect, so the trapped sound wave gradually leaks out and becomes the sound you hear. This process of letting a standing sound wave emerge slowly as a traveling wave is typical of woodwind and brass instruments. The reflection at an open pipe end depends on the shape of that end. Flaring it into the horn shape common in brass instruments reduces the reflection and eases the transition from standing wave to traveling wave. That's why horns project sound so well.

Check Your Understanding #9: Air Guitar

Why does an acoustic guitar have a sound box?

Answer: The sound box transfers the vibrational energy of the strings to the air.

Why: Guitar strings are too narrow to push effectively on the air and emit sound. They do better by transferring their energy to the body of the acoustic guitar so that its flat surfaces can push on the air. An electric guitar avoids the need for a sound box by converting the string's vibrations directly into electric currents and from there into movements of an audio speaker.