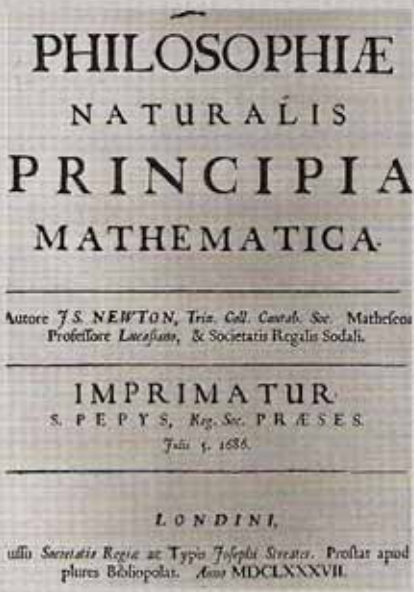
Newton's second and third laws

These laws are the cornerstone of what is called classical physics. They imply that, once the forces that act on a system are specified and the motion of the system is known at some point in time, then the motion of the system can be predicted at all future times. This predictability is characteristic of classical systems as opposed to quantum ones, where the uncertainty principle introduces a probabilistic interpretation on the future evolution of the system. Lately, this sharp definition of predictability has been eroded somewhat even for classical systems: chaotic behaviour can imply a loss of predictability in some cases.


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

This is the front page from Newton's book called *Principia* (Principles) in which he outlined his theories of the laws that governed the motion of objects.

Objectives

By the end of this chapter you should be able to:

- recognize situations of equilibrium, i.e. situations where *the net force and hence the acceleration are zero*;
- draw the forces on the body of interest and apply Newton's second law on that body, $F = ma$;
- recognize that the net force on a body is in the *same direction as the acceleration* of that body;
- identify pairs of forces that come from Newton's third law.

Newton's second law

This fundamental law asserts that:

▶ The net force on a body is proportional to that body's acceleration and is in the same direction as the acceleration. Mathematically

$$\vec{F} = m\vec{a}$$

where the constant of proportionality, m, is the *mass* of the body.

Figure 5.1 shows the net force on a freely falling body, which happens to be its weight, $W = mg$. By Newton's second law, the net force equals the mass times the acceleration, and so

$$mg = ma$$
$$\Rightarrow a = g$$

that is, the acceleration of the freely falling body is exactly g. Experiments going back to Galileo show us that indeed all bodies fall in a vacuum with the same acceleration (the acceleration due to gravity) irrespective of their density, their mass, their shape and the material from which they are made.

Figure 5.1 A mass falling to the ground acted upon by gravity.

▶ The equation $F = ma$ defines the unit of force, the newton (symbol N). One newton is the force required to accelerate a mass of 1 kg by 1 m s^{-2} in the direction of the force.

It is important to realize that the second law speaks of the net force on the body. Thus, if a number of individual forces act on a body, we must first find the net force by vector addition.

A simple everyday example of the second law is that when you jump from some height you bend your knees on landing. This is because by bending your knees you stretch out the time it takes to reduce your speed to zero, and thus your acceleration (deceleration) is least. This means that the force from the ground on to you is least.

Example questions

Q1

A man of mass $m = 70$ kg stands on the floor of an elevator. Find the force of reaction he experiences from the elevator floor when:

(a) the elevator is standing still;

(b) the elevator moves up at constant speed 3 m s^{-1};

(c) the elevator moves up with acceleration 4 m s^{-2};

(d) the elevator moves down with acceleration 4 m s^{-2}.

Answer

Two forces act on the man: his weight mg vertically down and the reaction force R from the floor vertically up.

(a) There is no acceleration and so by Newton's second law the net force on the man must be zero. Hence

$$R = mg$$
$$= 700 \text{ N}$$

(b) There is no acceleration and so again

$$R = mg$$
$$= 700 \text{ N}$$

(c) There is acceleration upwards. Hence

$$R - mg = ma$$

so

$$R = mg + ma$$
$$= 700 \text{ N} + 280 \text{ N}$$
$$= 980 \text{ N}$$

(d) We again have acceleration, but this time in the downward direction. Hence

$mg - R = ma$

so

$$R = mg - ma$$
$$= 700\text{ N} - 280\text{ N}$$
$$= 420\text{ N}$$

Note: In (c) the acceleration is up, so we find the net force in the upward direction. In (d) the acceleration is down, so we find the net force in the downward direction. Newton's law in all cases involves accelerations and forces in the same direction.

Q2

A man of mass 70 kg is moving *upward* in an elevator at a constant speed of 3 m s^{-1}. The elevator comes to rest in a time of 2 s. What is the reaction force on the man from the elevator floor during the period of deceleration?

Answer

The acceleration experienced by the man is -1.5 m s^{-2}. So

$$R - mg = ma$$
$$\Rightarrow R = mg + ma$$
$$= 700 + (-105) = 595\text{ N}$$

If, instead, the man was moving *downward* and then decelerated to rest, we would have

$$mg - R = ma$$
$$\Rightarrow R = mg - ma$$
$$= 700 - (-105) = 805\text{ N}$$

Both cases are easily experienced in daily life. When the elevator goes up and then stops we feel 'lighter' during the deceleration period. When going down and about to stop, we feel 'heavier' during the deceleration period. The feeling of 'lightness' or 'heaviness' has to do with what reaction force we feel from the floor.

Q3

A hot air balloon of mass 150 kg is tied to the ground with a rope (of negligible mass). When the rope is cut, the balloon rises with an acceleration of 2 m s^{-2}. What was the tension in the rope?

Answer

The forces on the balloon originally are its weight, the upthrust and the tension (see Figure 5.2).

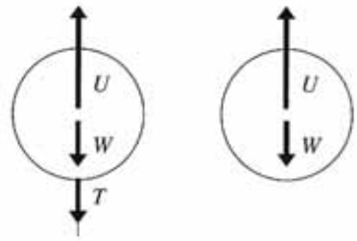


Figure 5.2.

Initially we have equilibrium and so $U = W + T$. After the rope is cut the net force is $U - W$ and so

$$U - W = ma$$
$$= 150 \times 2$$
$$= 300\text{ N}$$

From the first equation

$$T = U - W = 300\text{ N}$$

The next examples show how Newton's second law is applied when more than one mass is present.

Q4

Two blocks of mass 4.0 and 6.0 kg are joined by a string and rest on a frictionless horizontal table (see Figure 5.3). If a force of 100 N is applied horizontally on one of the blocks, find the acceleration of each block.

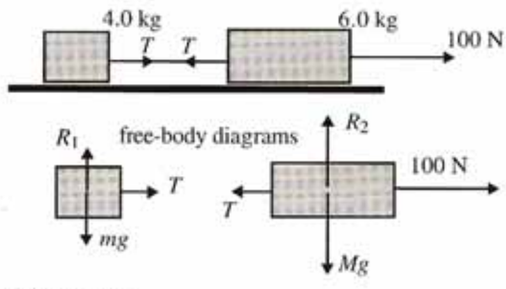


Figure 5.3.

Answer

Method 1: The net force on the 6.0 kg mass is $100 - T$ and on the 4.0 kg mass just T. Thus, applying Newton's second law separately on *each* mass

$$100 - T = 6a$$
$$T = 4a$$

Solving for a (by adding the two equations side by side) gives $a = 10\ \text{m s}^{-2}$ and the tension is thus

$$T = 4.0 \times 10$$
$$= 40\ \text{N}$$

Note: The free-body diagram makes it clear that the 100 N force acts *only* on the body to the right. It is a common mistake to say that the body to the left is also acted upon by the 100 N force.

Method 2: We may consider the two bodies as one of mass 10 kg. This is denoted by the dotted line in Figure 5.4. The net force on the body is 100 N. Note that the tensions are irrelevant now since they cancel out. (They did not in Method 1 as they acted on *different* bodies. Now they act on the *same* body. They are now *internal* forces and these are irrelevant.)

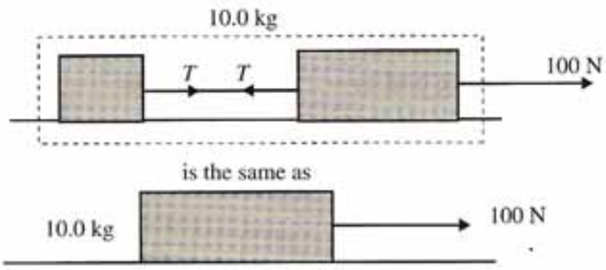


Figure 5.4.

Applying Newton's second law on the single body we have

$$100 = 10a$$
$$\Rightarrow a = 10\ \text{m s}^{-2}.$$

But to find the tension we must break up the combined body into the original two bodies. Newton's second law on the 4.0 kg body gives

$$T = 4a = 40\ \text{N}$$

(the tension on this block is the net force on the block). If we used the other block, we would see that the net force on it is $100 - T$ and so

$$100 - T = 6 \times 10$$
$$= 60$$

giving $T = 40$ N as before.

Q5

(Atwood's machine) Two masses of $m = 4.0$ kg and $M = 6.0$ kg are joined together by a string that passes over a pulley. The masses are held stationary and suddenly released. What is the acceleration of each mass?

Answer

Intuition tells us that the larger mass will start moving downward and the small mass will go up. So if we say that the larger mass's acceleration is a, then the other mass's acceleration will also be a in magnitude but, of course, in the opposite direction. The two accelerations are the same because the string cannot be extended.

Method 1: The forces on each mass are weight mg and tension T on m and weight Mg and tension T on M (see Figure 5.5).

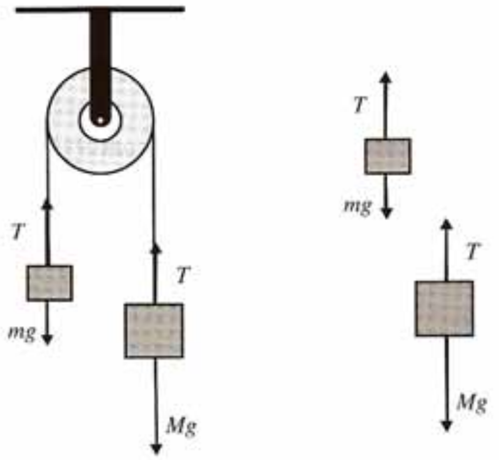


Figure 5.5.

Newton's second law applied to each mass states

$$T - mg = ma \quad (1)$$
$$Mg - T = Ma \quad (2)$$

Note these equations carefully. Each says that the net force on the mass in question is equal to that mass times that mass's acceleration. In the first equation we find the net force in the upward direction, because that is the direction of acceleration. In the second we find the net force downward, since that is the direction of acceleration in that case. We want to find the acceleration, so we simply add up these two equations side by side to find

$$Mg - mg = (m + M)a$$

hence

$$a = \frac{M - m}{M + m} g$$

(Note that if $M \gg m$, the acceleration tends to g (why?).) This shows clearly that if the two masses are equal then there is no acceleration. This is a convenient method for measuring g: Atwood's machine effectively 'slows down' g so the falling mass has a much smaller acceleration from which g can then be determined. Putting in the numbers for our example we find $a = 2.0 \text{ m s}^{-2}$. Having found the acceleration we may, if we wish, also find the tension in the string, T. Putting the value for a in formula (1) we find

$$\begin{aligned} T &= m\frac{M - m}{M + m} g + mg \\ &= 2\frac{Mm}{M + m} g \\ &= 48 \text{ N} \end{aligned}$$

(If $M \gg m$, the tension tends to $2mg$ (why?).)

Method 2: We treat the two masses as one body and apply Newton's second law on this body (but this is trickier than in the previous example) – see Figure 5.6.

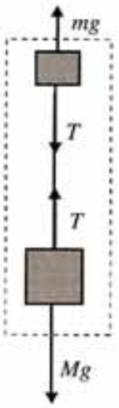


Figure 5.6.

In this case the net force is $Mg - mg$ and, since this force acts on a body of mass $M + m$, the acceleration is found as before from $F = \text{mass} \times \text{acceleration}$. Note that the tension T does not appear in this case, being now an internal force.

Q6

In Figure 5.7, a block of mass M is connected to a smaller mass m through a string that goes over a pulley. Ignoring friction, find the acceleration of each mass and the tension in the string.

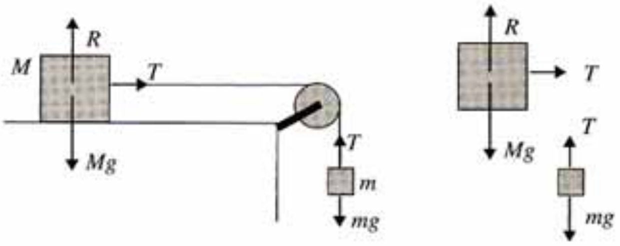


Figure 5.7.

Answer

Method 1: The forces are shown in Figure 5.7. Thus

$$\begin{aligned} mg - T &= ma \\ T &= Ma \end{aligned}$$

from which (adding the two equations side by side)

$$a = \frac{mg}{m + M}$$

(If $M \gg m$ the acceleration tends to zero (why?).) If $M = 8.0$ kg and $m = 2.0$ kg, this gives $a = 2.0 \text{ m s}^{-2}$. Hence

$$\begin{aligned} T &= \frac{Mmg}{m + M} \\ &= 16 \text{ N} \end{aligned}$$

Method 2: Treating the two bodies as one results in the situation shown in Figure 5.8.

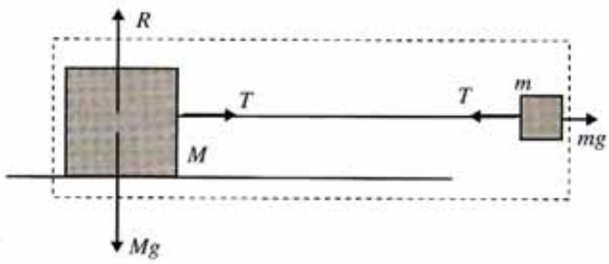


Figure 5.8.

The net force on the mass $M + m$ is mg. Hence

$$mg = (M + m)a$$

$$\Rightarrow a = \frac{mg}{m + M}$$

The tension can then be found as before.

Q7

Consider finally 100 blocks each of mass $m = 1.0$ kg that are placed next to each other in a straight line, as shown in Figure 5.9.

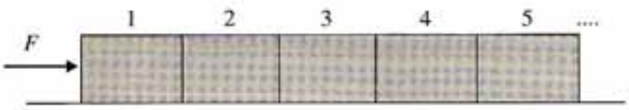


Figure 5.9.

A force $F = 100$ N is applied to the block at the left. What force does the 60th block exert on the 61st (see Figure 5.10)?

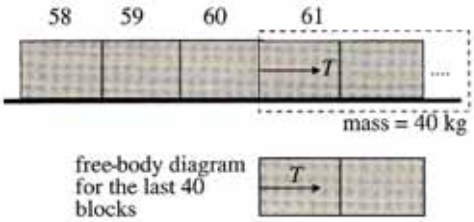


Figure 5.10.

Answer

To answer the question, we treat the 100 blocks as one body, in which case the net force on the system is 100 N. Since the mass is 100 kg, the acceleration of each block is 1 m s^{-2}.

Let T be the required force. It is the net force on the body inside the dotted line of mass 40 kg. Since this force accelerates a mass of 40 kg by 1 m s^{-2}, $T = ma = 40$ N.

Terminal velocity

When a body moves through a fluid (a gas or liquid), it experiences an opposing force that depends on the speed of the body. If the speed is small, the opposing force is proportional to the speed, whereas for larger speeds the force becomes proportional to the square of the speed. Consider, for example, a body falling through air. The forces on the body are its weight, mg, and the opposing force, which we assume is proportional to the speed, $F = kv$. Initially the speed is small, so the body falls with an acceleration that is essentially that due to gravity. As the speed increases, so does the opposing force and hence, after a while, it will become equal to the weight. In that case the acceleration becomes zero and the body continues to fall with a *constant* velocity, called *terminal velocity*. Figure 5.11 shows a body falling from rest and acquiring a terminal velocity of 50 m s^{-1} after about 25 s. The acceleration of the body is initially that due to gravity but becomes zero after about 25 s.

$$mg = kv_T \Rightarrow v_T = \frac{mg}{k}$$

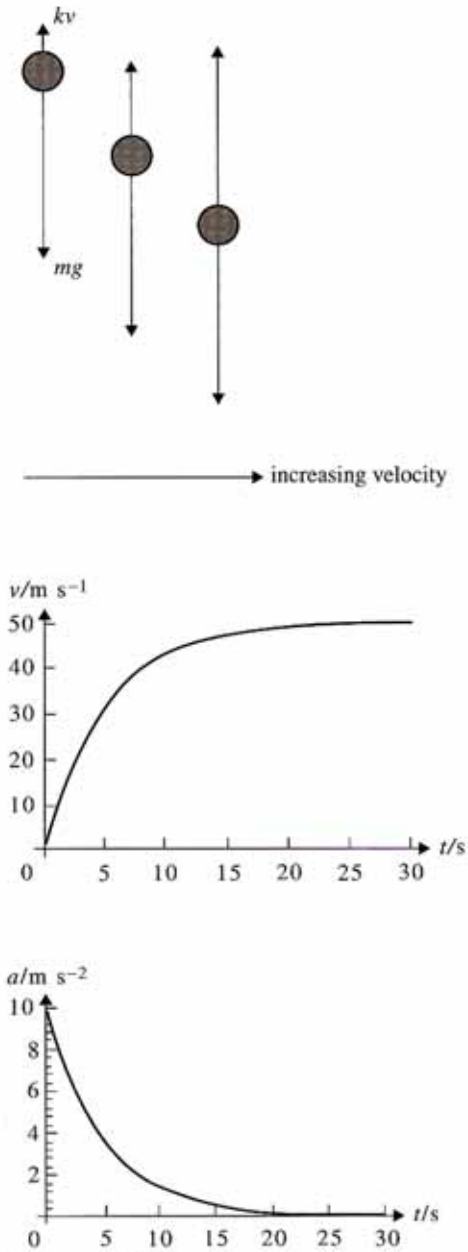


Figure 5.11 The opposing force grows as the speed increases and eventually becomes equal to the weight. From that point on, the acceleration is zero and the body has achieved its terminal velocity.

The inclined plane

The motion of a body along a straight line that is kept at an angle to the horizontal (inclined plane) is an important application of Newton's laws. The following is an example. A mass of $m = 2.0$ kg is held on a frictionless inclined plane of 30°. What is the acceleration of the mass if it is released?

There are two forces acting on the mass (see Figure 5.12).

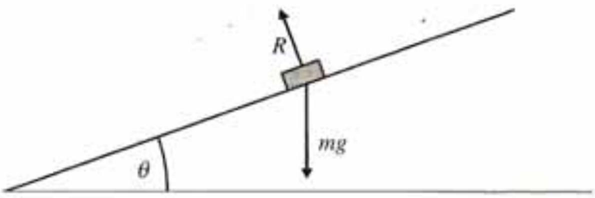


Figure 5.12 A mass on an inclined plane.

The two forces are: its weight vertically down of magnitude mg, and the reaction from the plane, which is perpendicular to the plane, of magnitude R. We can find the components of these two forces along two mutually perpendicular axes, one being along the plane (see Figure 5.13). The force R is already along one of the axes so we don't bother with that. But mg is not.

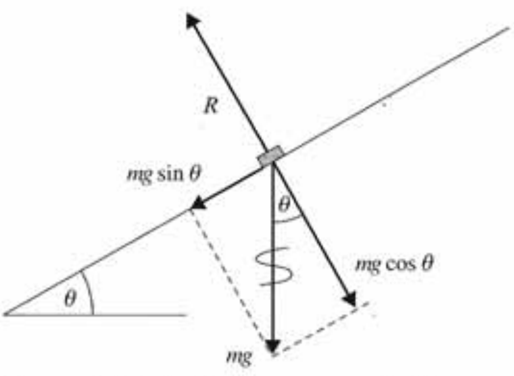


Figure 5.13 We take components along axes that lie along the plane and normally to the plane.

The magnitude of the component of mg along the plane is $mg \sin\theta$, where $\theta = 30°$ is the angle of the incline. This component lies down the plane. The other component is $mg \cos\theta$, in the direction perpendicular to the plane. In that direction there is no acceleration, hence the net force there is zero: that is, $R - mg\cos\theta = 0$. This tells us that $R = mg\cos\theta$. In the direction along the plane, there is a single unbalanced force, namely $mg \sin\theta$, and therefore by Newton's second law, this force will equal mass times acceleration in that same direction: that is

$$mg \sin\theta = ma$$

where a is the unknown acceleration. Hence, we find

$$a = g \sin\theta$$

This is an important result and we will make use of it many times. Note that the acceleration does not depend on the mass. For the numerical values of this problem we find $a = 5.0 \text{ m s}^{-2}$.

You may wonder why we took as our axes the ones along and perpendicular to the inclined plane. The answer is that we did not have to choose these axes. Any other set would have done. This choice, however, is the most convenient, because it exploits the fact that the acceleration will take place along the plane, so we choose that direction as one of our axes. If we had chosen another set of axes, say a horizontal and a vertical one, then we would find acceleration along both of these axes. Acceleration, of course, is a vector and if we combined these two accelerations, we would find the same acceleration (in magnitude and direction) as above. Try to work out the details.

An accelerometer

Consider a mass that is hanging from a string of length L, which is attached to the ceiling of a train. What will be the angle the string makes with the vertical if (a) the train moves forward with a constant speed of 3 m s^{-1}, or (b) moves forward with an acceleration of 4 m s^{-2}? If the train moves with constant speed in the horizontal direction, the acceleration in this direction is zero. Hence the net force in the horizontal direction must also be zero. The only forces on the mass are its weight vertically down, and the tension T of the string along the string. So, to produce zero force in the horizontal direction the string must be vertical. In case (b) there is acceleration in the horizontal direction and hence there must also be a net force in this direction. The string will therefore make an angle θ with the vertical (see Figure 5.14).

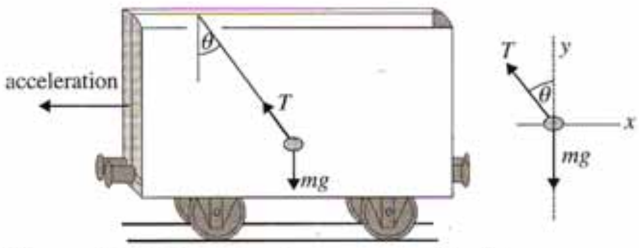


Figure 5.14 When the forces are not in the direction of acceleration, we must take components.

In this case we take components of the forces on the mass along the horizontal and vertical directions. In the horizontal direction we have only the component of T, which is $T \sin\theta$, and in the vertical direction we have $T \cos\theta$ upward and mg downward. Therefore

$$T \sin\theta = ma \quad \text{and} \quad T \cos\theta = mg$$

Hence, $a = g \tan\theta$. Note that the mass does not enter into the expression for a. This is actually a crude device that can be used to measure acceleration – an accelerometer.

Newton's third law

Newton's third law states that:

▶ If body A exerts a force F on body B, then body B exerts an equal but opposite force on body A. (See Figure 5.15.)

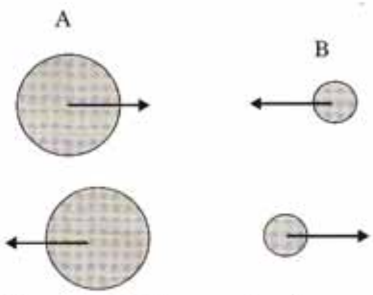


Figure 5.15 The two bodies exert equal and opposite forces on each other..

Make sure you understand that these equal and opposite forces act on *different* bodies. Thus, you cannot use this law to claim that it is impossible to ever have a net force on a body because for every force on it there is also an equal and opposite force. Here are a few examples of this law:

- You stand on roller-skates facing a wall. You push on the wall and you move away from it. This is because you exerted a force on the wall and in turn the wall exerted an equal and opposite force on you, making you accelerate away.
- You are about to step off a boat onto the dock. Your foot exerts a force on the dock, and in turn the dock exerts a force on you (your foot) in the opposite direction making you (and the boat) move away from the dock. (You probably fall in the water!)
- A helicopter hovers in air. Its rotors exert a force downward on the air. Thus, the air exerts the upward force on the helicopter that keeps it from falling.
- A book of mass 2 kg is allowed to fall freely. The earth exerts a force on the book, namely the weight of the book of about 20 N. Thus, the book exerts an equal and opposite force on the earth – a force upward equal to 20 N.

Be careful with situations where two forces are equal and opposite but have nothing to do with the third law. For example, a block of mass 3 kg resting on a horizontal table has two forces acting on it. Its weight of 30 N and the reaction from the table that is also 30 N. These two forces are equal and opposite, but they are acting on the same body and so have nothing to do with Newton's third law. (We have seen in the last bullet point above the force that pairs with the weight of the block. The one that pairs with the reaction force is a downward force on the table.)

Newton's third law also applies to cases where the force between two bodies acts at a distance: that is, the two bodies are separated by a certain distance. For example, two electric charges will exert an electric force on each other and any two masses will attract each other with the gravitational force. These forces must be equal and opposite. (See Figure 5.16.)

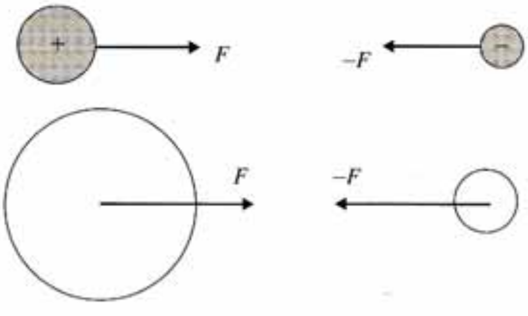


Figure 5.16 The two charges and the two masses are different but the forces are equal and opposite.

Questions

1 (a) Under what circumstances would a constant force result in an increasing acceleration on a body?
(b) Under what circumstances would a constant force result in zero acceleration on a body?

2 A car of mass 1354 kg finds itself on a muddy road. If the force from the engine pushing the car forward exceeds 575 N, the wheels slip (i.e. they rotate without rolling). What is the maximum acceleration that the car can move with on this road?

3 The net force on a mass of 1.00 kg initially at rest is 1.00 N and acts for 1.00 s. What will the velocity of the mass be at the end of the 1.00 s interval of time?

4 A mass of 2.00 kg is acted upon by two forces of 4.00 N and 10.0 N. What is the smallest and largest acceleration these two forces can produce on the mass?

5 A man of mass m stands in an elevator. Find the reaction force from the elevator floor on the man when:
(a) the elevator is standing still;
(b) the elevator moves up at constant speed v;
(c) the elevator accelerates down with acceleration a;
(d) the elevator accelerates down with acceleration $a = g$.
(e) What happens when $a > g$?

6 A bird is in a glass cage that hangs from a spring scale. Compare the readings of the scale in the following cases.
(a) The bird is sitting in the cage.
(b) The bird is hovering in the cage.
(c) The bird is moving upward with acceleration.
(d) The bird is accelerating downward.
(e) The bird is moving upward with constant velocity.

7 Get in an elevator and stretch out your arm holding your heavy physics book. Press the button to go up. What do you observe happening to your stretched arm? What happens as the elevator comes to a stop at the top floor? What happens when you press the button to go down and what happens when the elevator again stops? Explain your observations carefully using the second law of mechanics.

8 A block of mass 2.0 kg rests on top of another block of mass 10.0 kg that itself rests on a frictionless table (see Figure 5.17). The largest frictional force that can develop between the two blocks is 16 N. Calculate the largest force with which the bottom block can be pulled so that both blocks move together without sliding on each other.

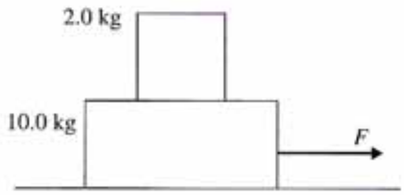


Figure 5.17 For question 8.

9 Figure 5.18 shows a person in an elevator pulling on a rope that goes over a pulley and is attached to the top of the elevator. Identify all the forces shown and for each find the reaction force (according to Newton's third law). On what body does each reaction force act?

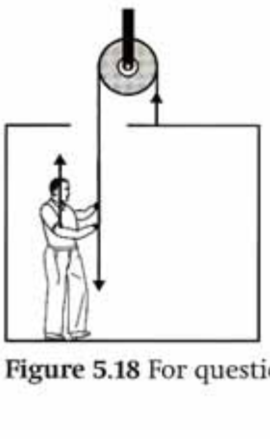


Figure 5.18 For questions 9 and 10.

10 Take the mass of the elevator shown in Figure 5.18 to be 30.0 kg and that of the person to be 70.0 kg. If the elevator accelerates upwards at 0.500 m s^{-2}, find the reaction force on the person from the elevator floor.

11 A small passenger car and a fully loaded truck collide head-on. Which vehicle experiences the greater force?

12 What force does a man of mass 80.0 kg exert on the earth as he falls freely after jumping from a table 1 m high from the surface of the earth?

13 Three blocks rest on a horizontal frictionless surface, as shown in Figure 5.19. A force of 20.0 N is applied horizontally to the right on the block of mass 2.0 kg. Find the individual forces acting on each mass. Identify action–reaction pairs.

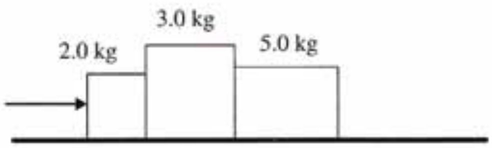


Figure 5.19 For question 13.

14 A (massless) string hangs vertically from a support in the ceiling. A mass of 10.0 kg is attached to the other end of the string. What is the force the string exerts on the support?

15 A block of mass 15.0 kg rests on a horizontal table. A force of 50.0 N is applied vertically downward on the block. Calculate the force that the block exerts on the table.

16 A block of mass 10.0 kg rests on top of a bigger block of mass 20.0 kg, which in turn rests on a horizontal table (see Figure 5.20). Find the individual forces acting on each block. Identify action–reaction pairs according to Newton's third law.

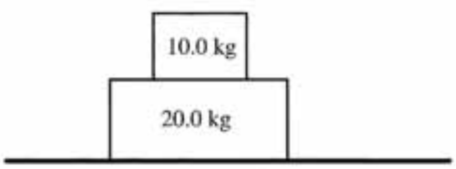


Figure 5.20 For question 16.

17 If a vertical downward force of 50.0 N acts on the top block in Figure 5.20, what are the forces on each block now.

18 A massless string has the same tension throughout its length. Can you explain why?

19 Look back at Figure 5.18. The person has a mass of 70.0 kg and the elevator a mass of 30.0 kg. If the force the person exerts on the elevator floor is 300.0 N, find the acceleration of the elevator ($g = 10$ m s^{-2}).

20 (a) Calculate the tension in the string joining the two masses in Figure 5.21.

(b) If the position of the masses is interchanged, will the tension change?

Figure 5.21 For question 20.

21 One hundred equal masses $m = 1.0$ kg are joined by strings as shown in Figure 5.22. The first mass is acted upon by a force $F = 100$ N. What is the tension in the string joining the 60th mass to the 61st?

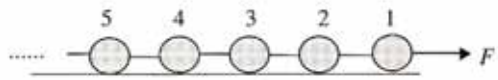


Figure 5.22 For question 21.

22 A mass of 3.0 kg is acted upon by three forces of 4.0 N, 6.0 N and 9.0 N and is in equilibrium. Convince yourself that these forces can indeed be in equilibrium. If the 9.0 N force is suddenly removed, what will the acceleration of the mass be?

23 What is the tension in the string joining the two masses in Figure 5.23? What is the acceleration of each mass?

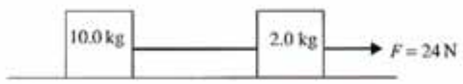


Figure 5.23.

24 Two bodies are joined by a string and are pulled up an inclined plane that makes an angle of 30° to the horizontal, as shown in Figure 5.24. Calculate the tension in the string when:

(a) the bodies move with constant speed;

(b) the bodies move up the plane with an acceleration of 2.0 m s^{-2}.

(c) What is the value of *F* in each case?

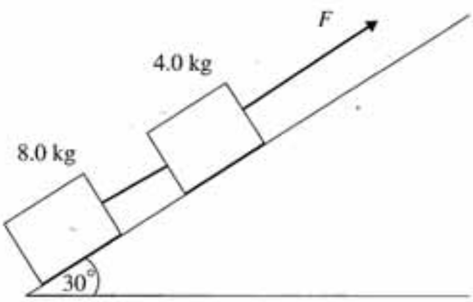


Figure 5.24 For question 24.

25 The velocity–time graph in Figure 5.25 is a student's graph for the vertical motion of a person who jumps from a helicopter and a few seconds later opens a parachute.

(a) Using the laws of mechanics *carefully* explain the shape of the curve. (When does the parachute open? When does the air resistance force reach its maximum value? Is the air resistance force constant?)

(b) How would you improve on the student's graph?

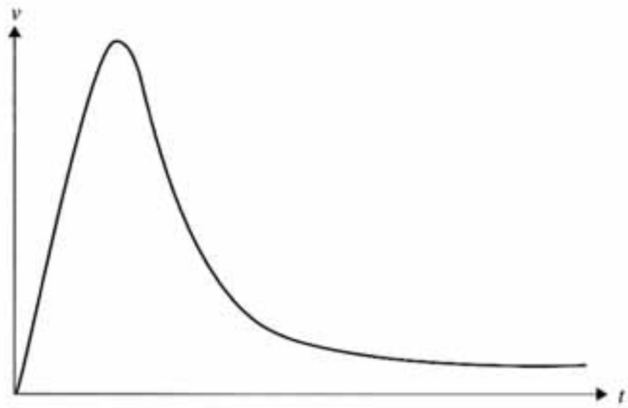


Figure 5.25 For question 25.