How Old are Stars

{**Abstract:** *In this chapter of the "How Old Is It" video book, we'll cover stars – second generation stars. We start with giant molecular clouds and what may trigger their collapse into star clusters. We'll follow their size, density, and temperature as they move from hydrostatic equilibrium to fragments to stars. We'll then follow the collapse of a fragment capable of creating a star like our Sun. We'll cover how circumstellar disks around central massive objects are formed. We'll cover the Protostar phase and examine a few like the ones in the Eagle Nebula and Orion. We'll cover T-Tauri stars, their properties, and examine several including T Tauri itself, XZ Tauri, and others. We'll cover how these giant molecular clouds form Open and Globular clusters and how Field Stars like our Sun have left their starting clusters. We'll follow stars as they start burning hydrogen and migrate to the Main Sequence on the H-R Diagram. We'll take a deep dive into the hydrogen fusion that powers stars. With a focus on our Sun, we'll compute proton collision and fusion rates. In order to understand the mass-luminosity relationship better, we'll cover the Coulomb Barrier and how Tunneling through it works. And finally, we'll examine what happens to a star once it runs out of hydrogen fuel.*}

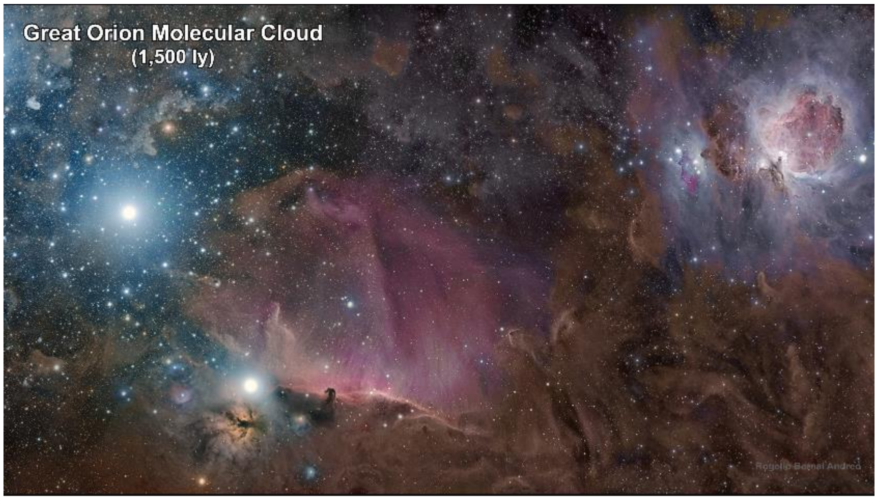
Introduction

According to the lambda Cold Dark Matter Big Bang theory, the Universe is around 13.8 billion years old. First generation stars would have contained hydrogen and a little helium and virtually nothing else because that is all that existed at the time they formed. [The formation process itself was driven by the 'caustics' process much like light coalesces at the bottom of a swimming pool.] Very few, if any of these methuselah first-generation stars are still around. What we see around us now are second generation stars including our own Sun. In this segment of the "How Old Is It" video book, we'll examine what we know about how old these stars are.

Molecular Cloud Collapse

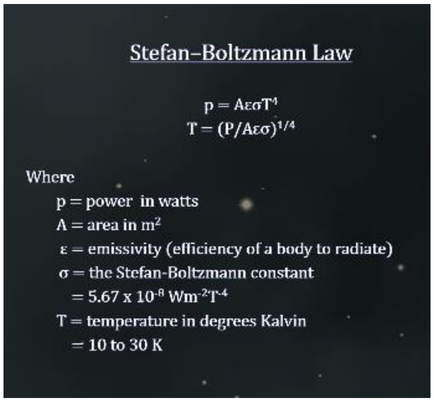
To understand how old a star is, we need to take a closer look at how stars form. We cannot observe the whole formation process for a single star. It takes millions of years. But we can learn a great deal by observing star formation at different stages starting with the molecular clouds that hold the matter that is transformed into the stars.

Thousands of giant molecular clouds exist in the disk of our galaxy. Each giant molecular cloud has 100,000's to a few million solar masses of material. We've seen some of these clouds in the Star Birth Nebula segment of the 'How far away is it' video book.

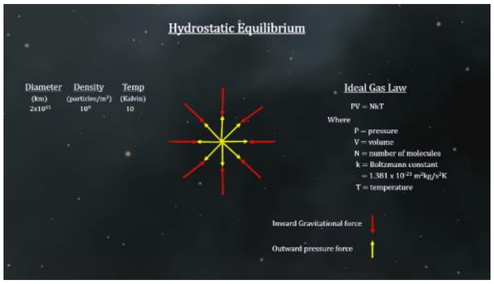


Giant Molecular clouds can be a large as 600 light years wide. This illustration is 200 light years in diameter. They contain mostly hydrogen and some helium. But they are also seeded with some heavier elements such as oxygen, carbon, iron and others from the dusty remains of earlier generation stars that ended as planetary nebula or super nova remnants. The clouds are dense relative to the rest of the gas between the stars (the interstellar medium) but are still much less dense than the atmosphere of a planet. Typical cloud densities are around a billion molecules per cubic meter. That might sound like a lot, but each cubic meter of air at the surface of the Earth has ten thousand trillion times more than that.

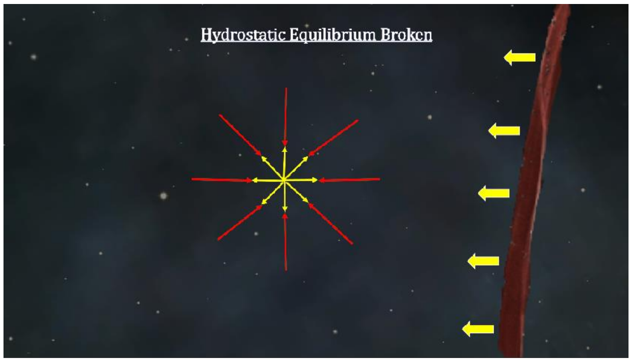
All bodies with a temperature greater than absolute zero radiate energy. Absolute zero is the temperature at which there is no molecular or atomic random motion. That would be 0 degrees Kelvin, -273° C or -460° F. To determine the temperature of a molecular cloud, we measure the incoming radiation power from a sizable area, and use the Stefan-Boltzmann Law to calculate the temperature. In our example we are using a cloud with 105 ly diameter. Results show that clouds like these are around 10 K. That's extremely cold. At these low temperature atoms combine to form molecules such as molecular hydrogen H_2, and water H_2O. Over 80 other molecules have been discovered in these clouds.



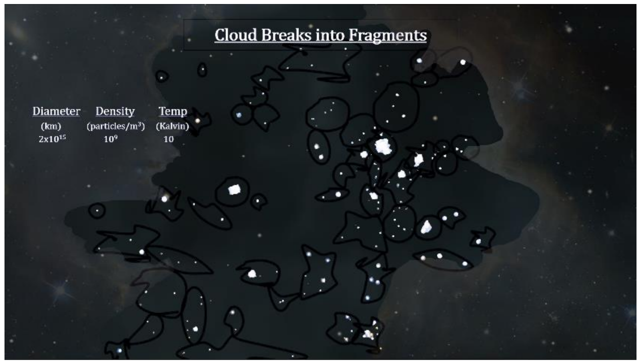
These clouds are in hydrostatic equilibrium and ruled by the ideal gas laws. At every point inside the cloud, the weak outward gas pressure is equal to the weak inward gravitational force.



We don't know what triggers a collapse. One thought is that it happens when clouds collide with each other. Another theory has it that supernova remnant wave fronts can do it. Here we see the remnant exert a force that compress a cloud to the point where it creates an imbalance in the gas pressure vs gravitational force in favor of the gravitational force. Another theory suggests that collapse is triggered when a cloud passes into a galaxy's spiral arm. In any case, once started, the collapse becomes extremely chaotic. And the chaos will continue until a new hydrostatic equilibrium is established.

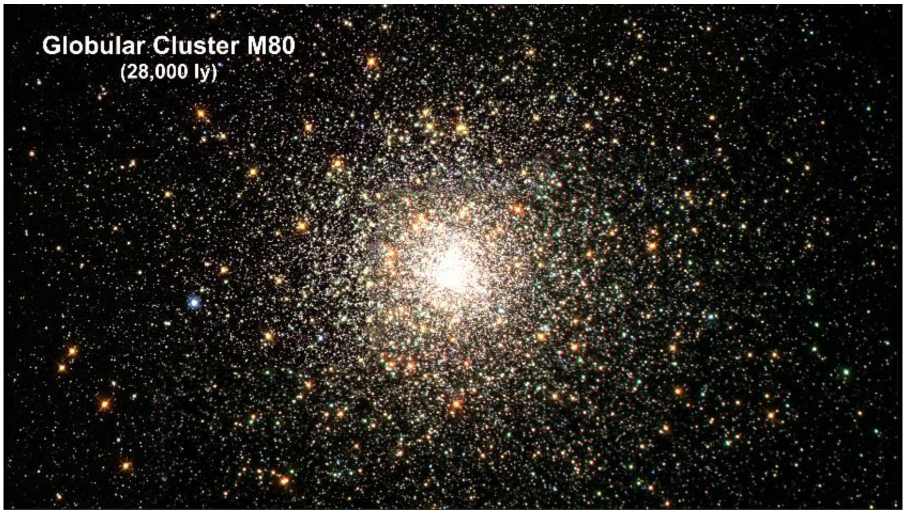


In our example, the cloud has compressed to a 180 ly diameter. Observations and computer simulations indicate that such a compression would lead to the cloud breaking up into fragments of various sizes and shapes within 2 million years. These fragments continue to collapse. Over a twenty million year period, they form planet sized objects, brown dwarfs, and stars of all masses. In this way, collapsing giant molecular clouds create star clusters.



The new hot stars radiation and shock waves push away lighter surrounding gas and dust and illuminate denser surroundings creating a sight like this one. NGC 603 is at the center of the star birth emission nebula N90 in the Small Magellanic Cloud orbiting the Milky Way. Hubble detected over 5,600 stars. A key point to remember for our 'how old is it' purposes is that these molecular cloud collapses always create star clusters with star counts that range from a few to hundreds of thousands of stars depending on the amount of matter in the original collapsing molecular cloud. We never see single stars being formed. All the stars in the cluster will be approximately the same age.

It will be a globular cluster like M80 if the gravitational force due to the total mass of all the stars is enough to bind them together.

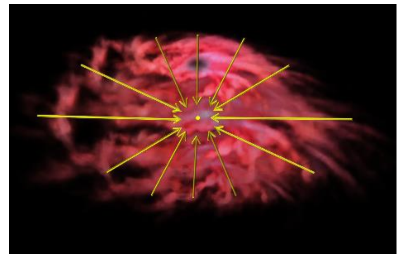


Or it will be an open star cluster, like the Pleiades. These stars still have some of their cloud fragment material in their vicinity. The Pleiades stars create this reflection nebula. These stars are all the same age. The fact that they are still close together, indicates and they are very young. Over the next two hundred and fifty million years these stars will drift so far apart that observers viewing these stars at that time won't be able to link them together.

Stars not associated with any cluster are called 'Field Stars'. Just stars in a field of stars. Our Sun is a field star. It has drifted far from the cluster it was formed in.

Star Formation

Now let's consider a fragment large enough to create a star the size of our Sun. The primary force on the rotating fragment is gravity pulling towards the center.



But because the subset of dust and gas that's rotating around the center of gravity follows the conservation of angular momentum laws, matter there will increase its velocity as its distance from the center decreases. This effectively slows this region's collapse into the center. In fact, if the velocity of a small piece reached orbital speeds, it would not fall into the forming star at all.

Angular Momentum

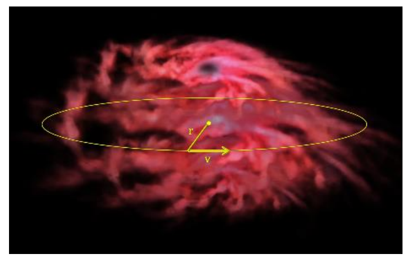
$L = mvr$

Where
- L = angular momentum
- m = mass of cloud particle
- v = velocity of cloud particle
- r = distance to the center

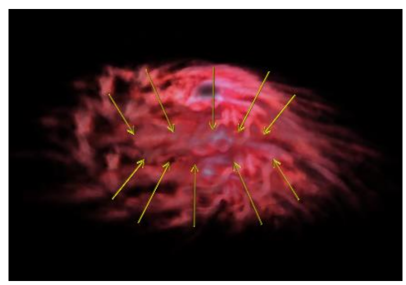
Conservation of Angular Momentum

L is constant

Therefore, if
r goes down,
v must go up



The overall effect is for the matter above the plane of rotation to move down and for the matter below the plane of rotation to move up. The entire fragment morphs into a disk structure around the core called a circumstellar disk.



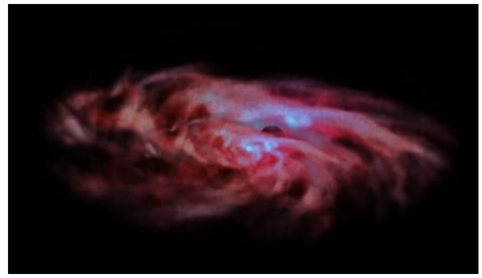
In the early stages of the collapse into the core, the temperatures remain low because generated radiation energy is able to escape. The compression continues until the core is dense enough to hold the energy. This takes around 30 thousand years.

Diameter (km)	Density (particles/m³)	Temp (Kalvin)
10^{12}	10^{12}	100

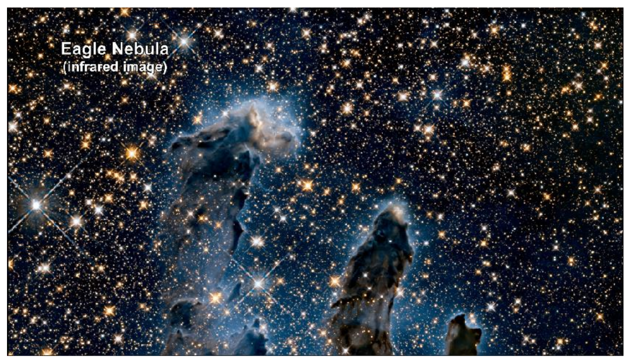
At that point, the core has enough mass density to capture generated photons and the temperature begins to raise dramatically. Another 100 thousand years and the core temperature reaches 10 thousand degrees Kalvin. At this temperature, the object begins to shine via normal (non-nuclear) means. It's now a protostar. So far, the core has only about 1% of its final mass.

Diameter (km)	Density (particles/m³)	Temp (Kalvin)
10^{10}	10^{18}	10,000

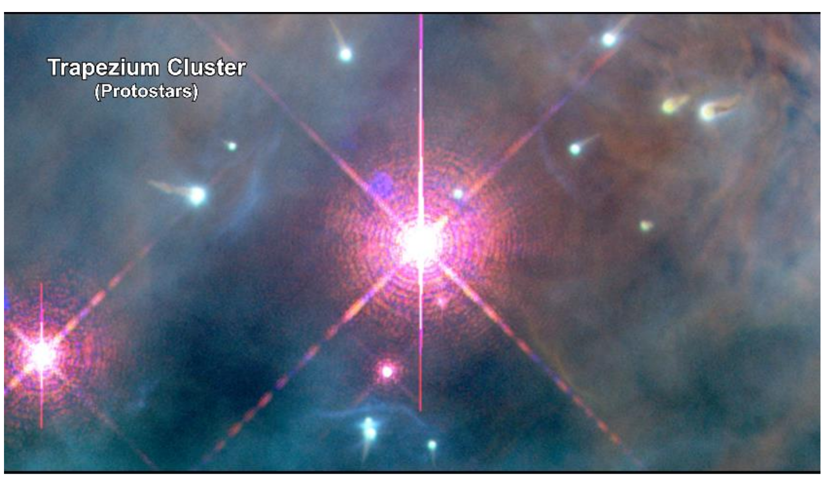
Matter accretes into the central object via this disk. Stars develop out of these fragments in three observable phases: protostars, pre-main sequence stars and main sequence stars. We can see stars in all three of these phases of their life cycle. [If a phase lasts long, we'll see lots of examples. If a phase is short-lived, we'll see only a few.]



Protostars remain shrouded in the dust and gas clouds that created them. The actual protostars can only be seen by infrared telescopes. The Eagle Nebula (M16) is a good example of this. It contains large numbers of forming stars. Some of these protostars can be seen with Hubble's near infrared image.



Here we see the Trapezium cluster of four stars at the heart of the Orion nebula. If we zoom in, we can see several protostars around one of the larger stars. They are the white objects streaming material away from the central star.

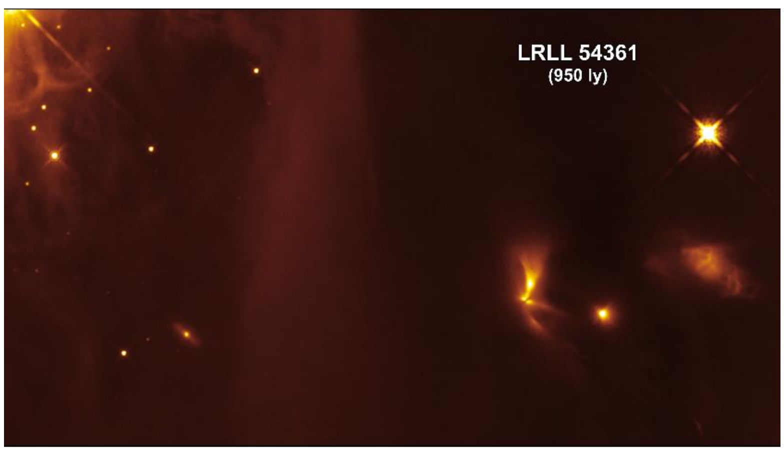


HBC 1 - 2700 ly

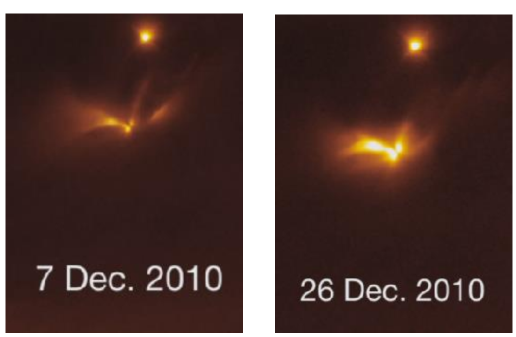
HBC 1 is another example. In this view, it illuminates a wispy reflection nebula.

Protostar LRLL 54361 – 950 ly

Here's one more example. This infrared Hubble Space Telescope image shows a protostar just 950 light years away. The protostar is the bright object with fan-like beams of light Flowing from it.



It is letting off flashes of light every 25.3 days. This time-lapse movie shows a pulse of light emanating from the protostar. Most if not all of this light is being scattered off the circumstellar envelope. The observational evidence along with computer simulations indicate that the protostar phase can last up to a few tens of millions of years.



T Tauri – 400 ly

Continuing to accumulate mass, its gravitational collapse causes the protostar's diameter to shrink significantly, and its core temperatures raise to 5 million K. Some astronomers point out that this is hot enough for some hydrogen fusion in its core. This makes it a true star. But because it is still growing by accreting large amount of material from its surroundings, it is not yet stable. Astronomers call these objects 'pre-main sequence stars' or 'young stellar objects. During this phase, a strong solar wind forms pushing back on the gas and dust surrounding it. A typical transition for a star with 3 times the mass our Sun or less is through a T-Tauri phase named after the star T Tauri.

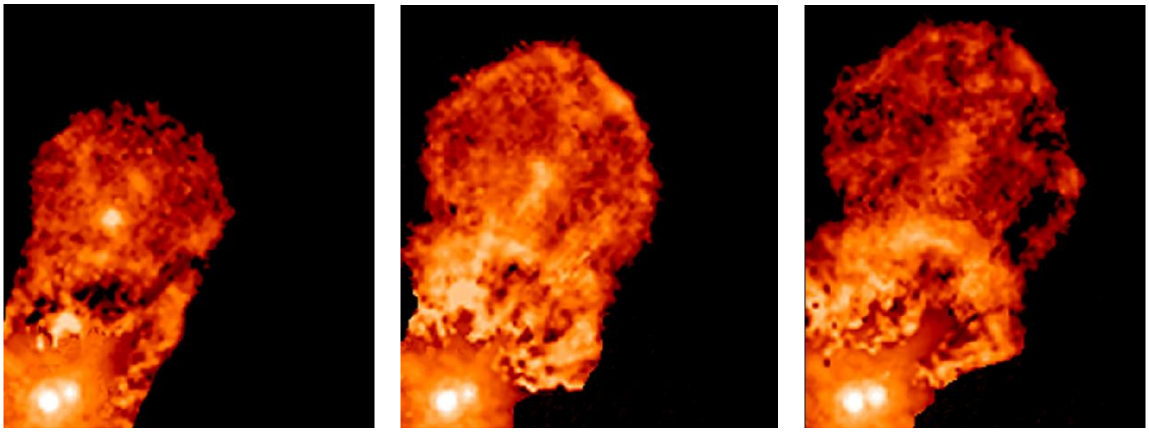
The orange star at the center of this photograph is T Tauri. It's the prototype of the T Tauri class of variable stars. Here we see it surrounded by a dusty yellow cosmic cloud named the Hind's Variable Nebula (NGC 1555/1554).

A typical characteristic of T-Tauri stars are jets of high-speed gas and dust streaming from both poles as strong magnetic fields guide matter from the star's circumstellar disk into the core. We can see these jets in star birth nebula. Here's a striking example from the Carina nebula. You can see the jets at the top of this Mystic Mountain.

XZ Tauri – 450 ly

Here's a view of a multiple star system called XZ Tauri, its neighbor HL Tauri and V1213 Tauri just 450 light years away. These young stellar objects are illuminating the entire region.

XZ Tauri is actually a binary star system. It is expelling hot bubbles of gas into the surrounding space. Gas from an unseen disk around one or both of the stars is channeled through magnetic fields surrounding the binary system and forced out into space at nearly 540,000 kilometers per hour (that's 300, 000 miles per hour). This outflow, which is only about 30 years old, extends nearly 96 billion kilometers or 60 billion miles from the star.



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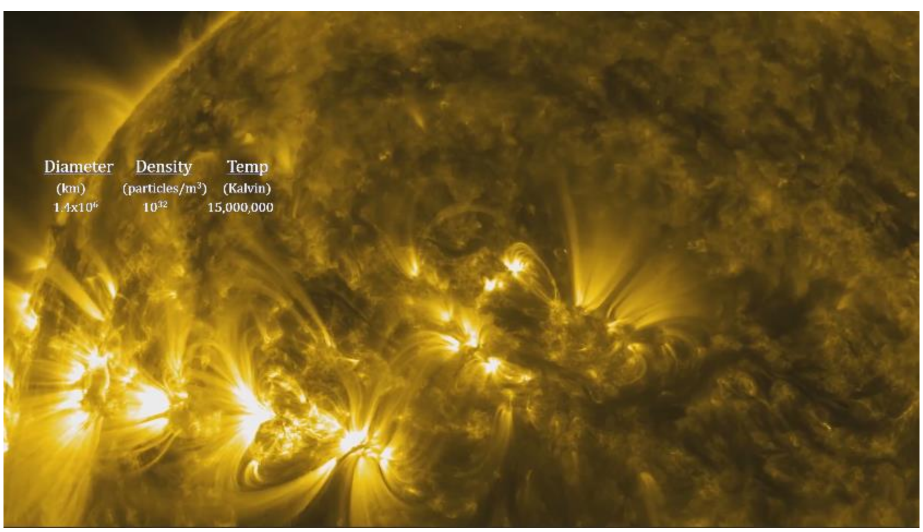
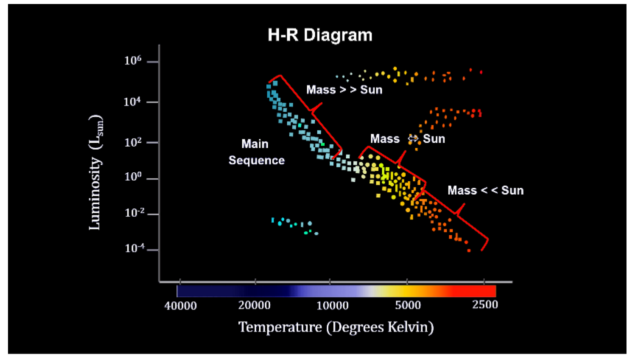
Stars can remain in the T-Tauri phase for as long as 100 million years and reach 10 million K at their cores. As their solar winds picks up, they disperse the remaining gas and dust around them back into the interstellar medium. This ends mass accumulation, and the stars settle into hydrostatic equilibrium. At the mass of our Sun, the core temperature reaches 15 million degrees K and it has 99.8 percent of all the collapsing molecular cloud fragment's matter, leaving only 0.2 % left over for plants, moons, asteroids, and comets. It is now a main sequence star.



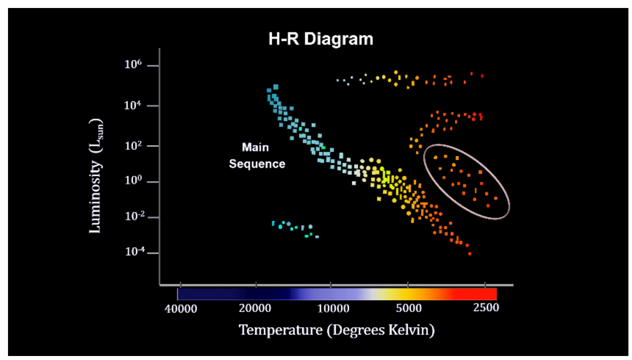
Main Sequence Star Lifetimes

Here's the Hertzsprung-Russell or H-R Diagram we covered in the "Distant Stars" segment of the "How far away is it" video book. The long diagonal line represents the main sequence for stars in hydrostatic equilibrium burning hydrogen.

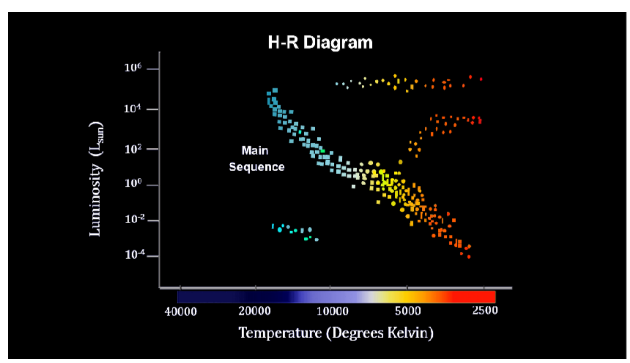
The lower right red stars are cool low mass stars that are a fraction of the mass of the Sun. The middle yellow and orange stars are closer to the mass of the Sun. And the upper left blue and white stars are hot high mass stars, many times more massive than the Sun. Protostars that are many times the mass of the Sun, evolve so rapidly, that they show up at the high blue end of the main sequence in a very short amount of time. For them, there is no T-Tauri phase.



T-Tauri stars would start here on the diagram. As they stabilize, they shrink in size, increase in temperature, start fusing hydrogen and migrate to the main sequence. The more massive the young stellar object, the higher up on the main sequence the eventual star will land. If we start counting the age of a star from the beginning of the cloud collapse that formed it, we would have stars just reaching the main sequence at around 150 to 200 million years old.

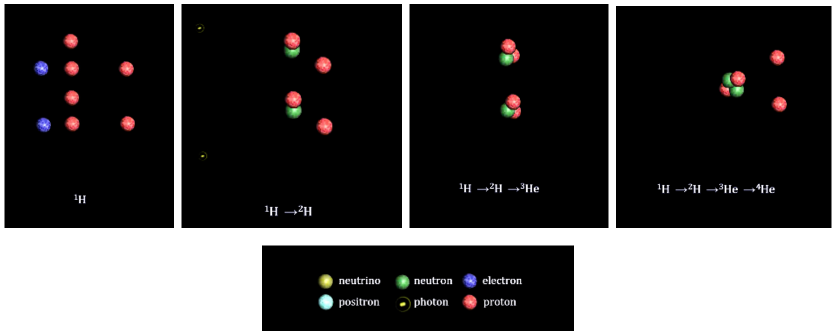


Stars will remain on the main sequence for as long as it takes to use up their hydrogen fuel. How long any particular star currently on the main sequence will remain there, depends on how much fuel it started with, how fast it is burning that fuel, and how long ago it stared to burn.



The mass of a star gives us a measure of how much fuel it started with, and its luminosity gives us a measure of how fast it is consuming this fuel. But before we can make the lifetime calculation, we need to understand how much energy we get from hydrogen when it fuses into helium. [Nuclear physicists cannot recreate the conditions inside the sun's core, but with particle accelerators like

CERN and nuclear reactors, they've learned a lot about the processes.] Here we are looking at the proton-proton fusion chain - the most common fusion reaction in our Sun's core.



The mass of Helium 4 at the end is a bit smaller than the mass of the four protons at the start. The amount of energy generated is determined by Einstein's equation $E = mc^2$. [98% of this energy is being carried away in the form of gamma ray photons that heat the core, and 2% is being carried away by neutrinos.]

Let

$E_{p\text{-}p}$ = energy released by the proton-proton fusion chain
m_p = mass of a proton = 1.6729×10^{-27} kg
m_{H4} = mass of a helium 4 nucleus = 6.6443×10^{-27} kg
m = mass converted into energy

We have

$$\begin{aligned} m &= 4m_p - m_{H4} \\ &= 4(1.6729 \times 10^{-27}) - 6.6443 \times 10^{-27} \\ &= 0.0473 \times 10^{-27}\ \text{kg} \end{aligned}$$

And

$$\begin{aligned} E_{p\text{-}p} &= mc^2 \\ &= (0.0473 \times 10^{-27}\text{kg}) \times (3 \times 10^8\ \text{m/s})^2 \\ &= 4.257 \times 10^{-12}\ \text{kg}\,\text{m}^2\text{s}^{-2} \\ &= 0.000000000004257\ \text{J} \end{aligned}$$

We see that the production of each helium nucleus releases only a small amount of energy. But by measurement, we know that the Sun produces 3.9×10^{26}. To produce this amount of energy, it would take a tremendous number of these fusion events every second.

Let

E = Energy released by fusing 4 hydrogen atoms into Helium 4 = 4.257×10^{-12} J
W = Energy released by the Sun per second = $3.90 \times 10^{26}\ \text{J}^{s-1}$
N = Number of p-p fusion chains per second

Then

$$\begin{aligned} N &= W/E \\ &= (3.90 \times 10^{26}\ \text{Js}^{-})/(4.257 \times 10^{-12}\text{J}) \\ &= 9.161 \times 10^{37}\ \text{p-p fusions per second} \\ &= 91{,}610{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \end{aligned}$$

We calculate that 613 million metric tons of hydrogen fuse to form 609 million metric tons of helium converting 4 million metric tons of matter into energy every second.

Let

N = Number of fusions per second = 9.161×10^{37}
m_p = mass of a proton = 1.6729×10^{-27} kg
m_{H4} = mass of a helium 4 nucleus = 6.6443×10^{-27} kg
M_H = mass of hydrogen fused per second
M_{He} = mass of helium produced per second
M_c = mass converted to energy per second

Then

$$\begin{aligned} M_H &= 4m_p \times N \\ &= (6.6916 \times 10^{-27}\ \text{kg}) \times 9.161 \times 10^{37} \\ &= 613 \times 10^9\ \text{kg} \\ M_{He} &= (6.6443 \times 10^{-27}\ \text{kg}) \times 9.161 \times 10^{37} \\ &= 609 \times 10^9\ \text{kg} \\ M_c &= M_H - M_{He} \\ &= 4 \times 10^9\ \text{kg} \end{aligned}$$

To figure out how long it would take for our sun to burn all the hydrogen it started with, we simply divide the available hydrogen by the amount consumed per second. The Sun's mass is 2 x 10^{30} kg. Fusion is only occurring in the core which represents about 10% of the Sun's mass. We see that once hydrogen burning began, our Sun would take 10 billion years before it ran out of fuel. That's the total amount of time the Sun will be a main sequence star. [At the end, it will expand and cool into a red giant star and consume the Earth.]

Let

$t_\odot$ = **total amount of time the Sun will burn hydrogen**

$M_\odot$ = **Mass of the Sun**

$= 2 \times 10^{30}$ **kg**

M_H = **mass of hydrogen fused per second**

$= 613 \times 10^{9}$ **kg**

1 year $= 3.15 \times 10^{7}$**s**

Then

$$t_\odot = (0.1 M_\odot) / M_H$$
$$= (0.1 \times 2 \times 10^{30}\ \text{kg})/(61.3 \times 10^{10}\ \text{kgs}^{-1})$$
$$= 0.003263 \times 10^{20}\text{s} \times 1\text{yr}/(3.15 \times 10^{7}\text{s})$$
$$= 10.36 \times 10^{9}\ \text{years}$$

Astronomers have empirically found that, even though the more massive stars have more hydrogen fuel, a corresponding dramatic increase in luminosity shows that they are consuming this fuel faster – much faster. A small change in mass leads to a small change in the core temperature, but a very large change in the luminosity.

Let

M = **mass**

R = **radius**

L = **luminosity**

T_c = **core temperature**

τ = **time on the main sequence**

m_p = **mass of a proton**

$= 1.673 \times 10^{-27}$ **kg**

k_B = **Boltzmann constant**

$= 1.381 \times 10^{-23}$ **J/K**

For the Sun

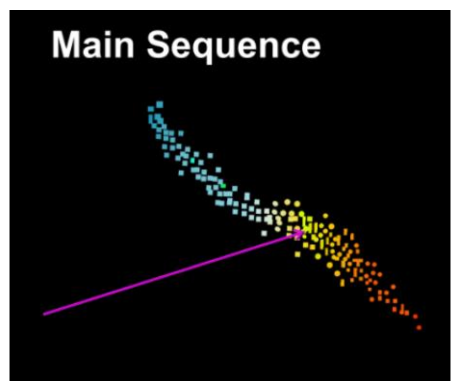
$M_\odot = 2 \times 10^{30}$ **kg**

$R_\odot = 6.96 \times 10^{8}$ **m**

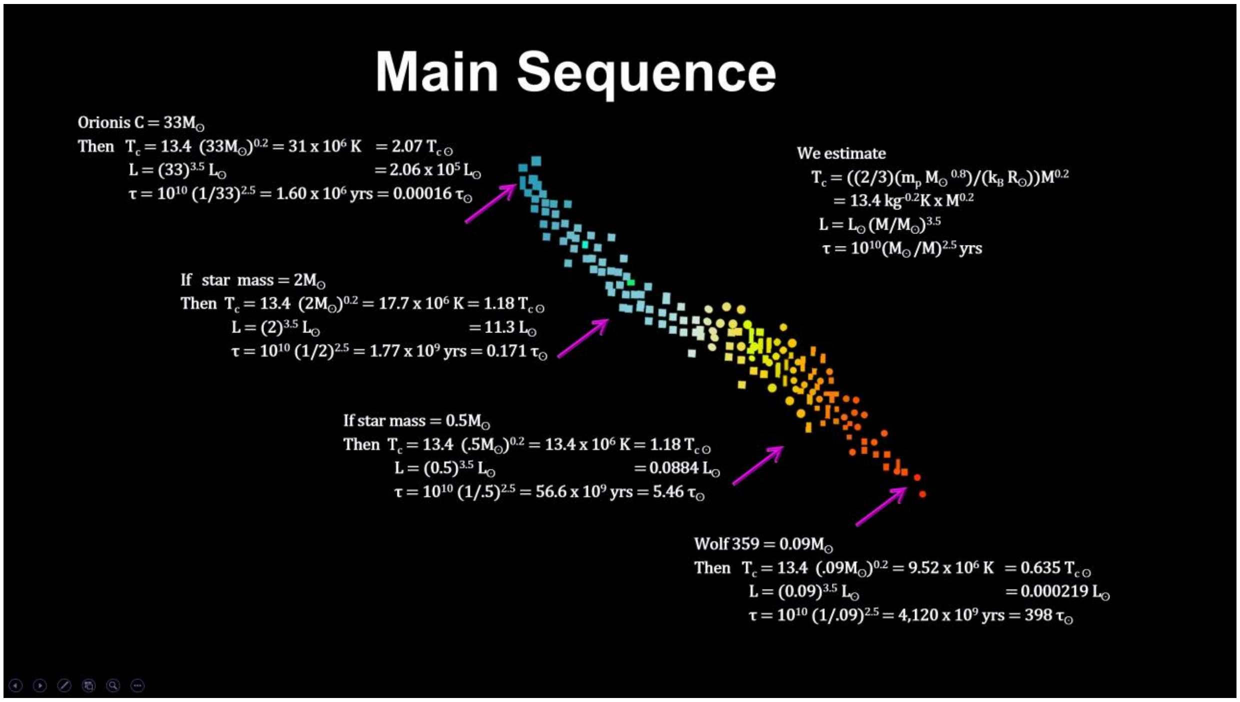
$L_\odot = 3.90 \times 10^{26}$ **J**

$T_{c\odot} = 15 \times 10^{6}$ **K**

$\tau_\odot = 10.36 \times 10^{9}$ **years**



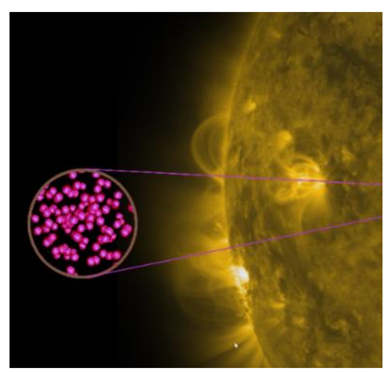
For example, stars twice the mass of the Sun, have over ten times the luminosity and burn out in under 2 billion years. In the other direction, we see that stars with half the mass of the Sun have less than a tenth the luminosity and remain on the main sequence five times longer. At the extremes, we have Orionis C1, in the Trapezium Cluster at 33 times the mass of the Sun. Its luminosity is over 200 thousand times greater than the Sun and it won't last more than a few million years. At the other end, Wolf 359 is just under a tenth of the mass of the sun and will remain a main sequence star almost 400 times longer than the sun.



The dramatically shorter time on the main sequence for higher mass stars and extremely extended time for lower mass stars is due to the proton fusion process's sensitivity to temperature. To understand why this is the case, we need to look at the proton-proton activity in the core one level deeper.

Solar Fusion Rates

Deep inside a star's core, protons are colliding at a tremendous rate. But few of these collisions result in a fusion of the two protons. That's because when protons collide, they have to overcome a repulsive electric force due to the fact that they both carry a positive charge. This is called the Coulomb Barrier. In order to understand why star luminosity is so sensitive to small increases in temperature, we need to see how this barrier is breached. We'll start with a look inside the Sun's core.



Using the known relationship between temperature and kinetic energy, we can calculate the average thermal energy and velocity of the protons in the Sun's core. It depends entirely on the temperature (15 million degrees K). We find that each proton has on average 2 kilo-electron volts of kinetic energy and travels at just over 600 thousand m/s. That's well over a million miles per hr.

Proton Thermal Energy

$$E = (3/2)kT$$

Where

E = thermal energy

T = temperature

k_B = Boltzmann constant = 1.381×10^{-23} J/K

With $T = 15 \times 10^6$ K

We get

$E = 3.1 \times 10^{-17}$ J = 2 keV

Proton Thermal Velocity

$$v_{th} = (3k_BT/m_p)^{1/2}$$

Where

T = temperature

m_p = mass of a proton = 1.673×10^{-27} kg

k_B = Boltzmann constant = 1.381×10^{-23} J/K

For the Sun

T = 15,000,000 K

And

$v_{th} = 6.09 \times 10^5$ m/s

With this, we can calculate the number of times a proton will collide with another proton. The number depends on the proton density, cross section, and thermal velocity. We cover 'cross section' in the "How small is it" video book. It represents the target area for determining a collision vs. a miss. We calculate that each proton experiences over one trillion collisions per second.

Proton Collision Rate

$$\Gamma = n_p \sigma v_{th}$$

With

Γ = collisions per second per proton

n_p = number of protons per cm^3 = 6×10^{25} cm^{-3} in our Sun's core

σ = proton cross section = 3.14×10^{-22} cm^2

v_{th} = proton thermal velocity = 6.09×10^5 m/s

We get

$\Gamma = 1.15 \times 10^{12}$

We can also calculate the Sun's fusion rate per proton. Dividing the mass of the sun's core by the mass of a proton gives us the number of protons in the core. Dividing the p-p fusion rate calculated earlier, by the number of protons in the core gives us the fusion rate per proton. And if we dived this by the trillion collisions each second, we get the number of fusions per collision. We see that the probability that any particular collision will result in a fusion is extremely small. That's why a proton's trillion collisions per second can go on for billions of years before one of them results in a fusion event.

Fusion Rate per Proton Collision

With

N = Number of fusions per second = 9.161×10^{37}

n_p = number of protons in the core

$M_\odot = 2 \times 10^{30}$ kg

$m_p = 1.673 \times 10^{-27}$ kg

$\Gamma = 1.15 \times 10^{12}$

We get

$n_p = M_\odot / m_p$

$= 0.1 \times 2 \times 10^{30}$ kg / 1.673×10^{-27} kg

$= 1.2 \times 10^{56}$

And the fusion rate per proton is

$N/n_p = 9.161 \times 10^{37} / 1.2 \times 10^{56}$

$= 7.63 \times 10^{-19}$

And the fusion rate per proton collision is

$(N/n_p)/\Gamma = 7.63 \times 10^{-19} / 1.15 \times 10^{12}$

$= 6.63 \times 10^{-31}$

Solar Coulomb Barrier

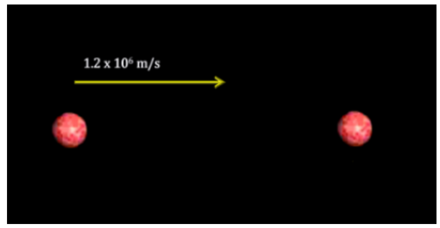
To understand why the vast majority of proton-proton collisions in our Sun don't result in fusion, even though they are colliding at incredible speeds, we need to examine the strength of the electrostatic force separating them. According to Coulomb's Law, two particles with the same charge will repel each other with a force that is proportional to the product of their charge and inversely proportional to the square of their distance from each other – very much like gravitational force.

Coulomb's Law

$F = k_C q_1 q_2 / r^2$

Where

- F = electro static force
- C = coulomb (units for charge)
- k_C = Coulomb's constant $= 8.99 \times 10^9$ Nm^{2}C^{-2}
- q_i = charges
- r = distance between charges



In order to fuse, the protons must get close enough for the attractive strong nuclear force to take over from the repulsive electric force. The reach of the strong force is very small - just over one femtometer or fermi. There are a million fermi's in a nanometer. At this distance, the electric repulsion force is overwhelming. We see that the force is quite extreme given that it is working on a mass as small as a single proton.

For two protons 1.2 + 1.2 = 2.4 fermis apart

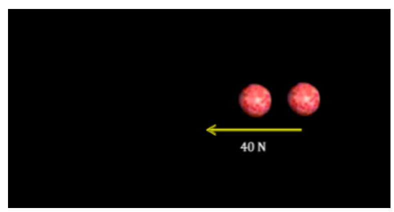
$q = +1.602 \times 10^{-19}$ C

$r = 2.4 \times 10^{-15}$ m

We get

$F = (23.1 \times 10^{-29}\ \text{Nm}^2) / (2.4 \times 10^{-15}\ \text{m})^2$

$= 40.1$ N $= 0.894$ pounds



If we look at it from a classical energy point of view, we see that proton average energy at 15,000,000 K is just not enough to overcome the potential energy barrier. In fact, the energy required to overcome the barrier is 300 times greater than the average energy of the protons.

Coulomb Barrier Energy

$U_0 = k_C q_1 q_2 / r$

Where

- F = electro static force
- C = coulomb (units for charge)
- k_C = Coulomb's constant $= 8.99 \times 10^9$ Nm^{2}C^{-2}
- q_i = charges
- r = distance between charges

For two protons 1.2 + 1.2 = 2.4 fermis apart

$q = +1.602 \times 10^{-19}$ C

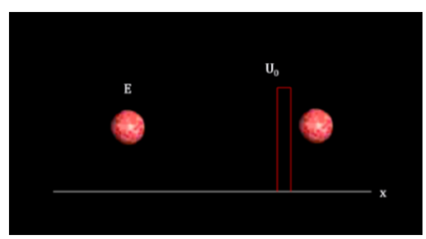
$r = 2.4 \times 10^{-15}$ m

We get

$U_0 = 9.59 \times 10^{-14}$ J = 600 keV

But proton energy is

$E = 3.1 \times 10^{-17}$ J = 2 keV



To understand how often we can expect a proton to have the barrier's energy at the Sun's temperature, we use probability distributions developed by James Maxwell and Ludwig Boltzmann in the mid-1800s. This analysis shows that only 1 out of 10^{200} collisions would cross the Coulomb Barrier. That's almost as good as none! Our Sun would simply not burn hydrogen, if this was all there was. But we know there is more, because we have measured the fusion rate for a proton in our Sun's core and it's a hundred and 80 orders of magnitude more than this classical physics predicts.

Maxwell-Boltzmann Distribution

$$f(E) = 2\,(E/\pi)^{1/2}\,(1/kT)^{3/2}\,e^{-(E/kT)}$$

Where

E = Coulomb barrier energy
$= U_0 = 9.59 \times 10^{-14}$J
$T = 1.5 \times 10^7$ K
k = Boltzmann's constant
$= 1.38 \times 10^{-23}$JK^{-1}

For $E >> kT$ we have

$f(E) \sim e^{-(E/kT)}$
$\sim e^{-(457)} = 1/e^{457} = 1/10^{200}$

Measured fusion rate = $7.63/10^{19}$

Tunneling through the Coulomb Barrier

At these extremely small distances, quantum mechanics plays a dominating role. A proton travels as a wave described by Schrodinger's equation. As a wave, its exact location is not completely knowable. The square of the particle's wave function gives us the probability for materializing as a particle at any particular location and time. Most of the time, it will be found at the most probable location.

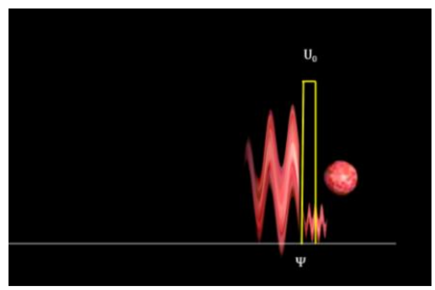
Free Particle Wave Function

$\Psi = Ae^{-\alpha x}$ where $\alpha = (2m(E - U_0)/\hbar^2)^{1/2}$

Ψ = is the wave function
A = wave amplitude
$\hbar$ = Planck's constant / 2π
m = mass
E = total particle energy
U_0 = barrier potential energy

For proton-proton fusion:
$m = m_p$
$U_0 = 600$ keV
$E = 3.1 \times 10^{-17}$ J = 2 keV

$|\Psi(x,t)|^2$ = the probability of finding the particle at x at time t



But we see that some of the wave function is on the far side of the barrier. The wave amplitude is significantly smaller, but the frequency is the same. The small amplitude indicates that there is as small probability that it will materialize there. But having the same frequency indicates that it will have the full particle energy. The phenomenon is called 'tunneling'. At the Sun's temperature, the probability that a proton will tunnel through the barrier is quite small – but a hundred and 93 orders of magnitude more likely than classical physics would have it.

Tunneling Probability

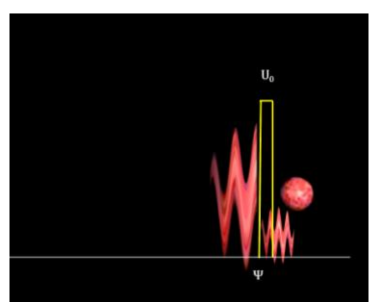
$P = e^{-a}$ where $a = -2\pi(Z_1^2 Z_2^2 q^4 M/2E\hbar)^{1/2}$

Where

P = the probability of barrier penetration
Z_i = particle atomic mass
q = particle charge
$\hbar$ = Planck's constant / 2π
M = reduced mass
E = total particle energy

For proton-proton fusion:
$M = m_p/2$
$Z_1 = Z_2 = 1$
$q = 1.602 \times 10^{-19}$ C
$E = 3.1 \times 10^{-17}$ J = 2 keV

We get
$P = e^{-16} = 10^{-7}$



Protons will cross the barrier and overlap many times before they actually trigger a fusion event. To be exact, converting probabilities to rates, we see that, on average, there are a million successful tunnelings through the Coulomb barrier to get one fusion.

Tunneling Rate

$P = 1 - e^{-\lambda}$

Where

P = the probability per unit time = e^{-16}

λ = rate per second

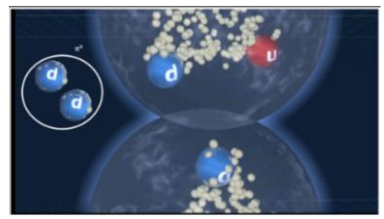
Which yields a tunneling rate = 1×10^{-9}

Measured proton fusion rate = 7.63×10^{-19}

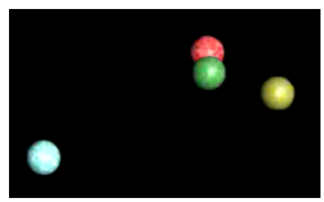
We have

tunnellings / fusions = $1 \times 10^{-9} / 7.63 \times 10^{-19} = 1.13 \times 10^{9}$

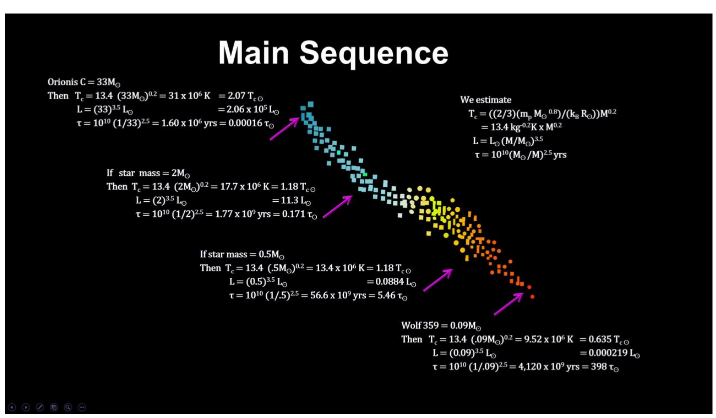
This is because, in order to fuse, a pion transfer between the two protons or something similar has to occur in the extremely short period of time that the protons are in contact. This particle sharing in the nucleus is similar to the electron sharing that binds molecules. We cover how this works in the "Higgs Boson" segment of the "How small is it" video book.



In addition, one of the protons has to eject of a neutrino and a positron to become a neutron. All these nuclear processes are sensitive to proton energies (i.e. temperature).



To conclude, we see that small increases in the colliding proton's energy (driven by increases in temperature) will increase: collisions, cross sections, coulomb barrier penetration rates, and fusion rates for overlapping protons. The cumulative effect makes the hydrogen burn rate exponentially sensitive to temperature.

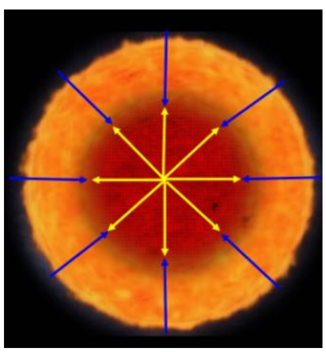


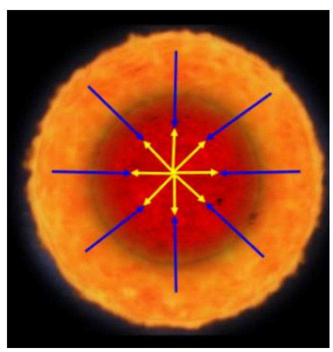
When Stars Run Out of Hydrogen

Now, given the mass of a star, we know how long it will burn the core hydrogen it started with. For the Sun, that's around 10 billion years. But how long the fuel will last from now, depends on how long ago it started fusing. Let's take a look at one way we know how far along a star is. In a star cluster, like M67 for example, if we can find a star that is just starting to show the symptoms of having run out of its core hydrogen, we'll know from its mass just how old it is. That will then be the age of all 1100 plus stars in the cluster, because they all formed at the same time.

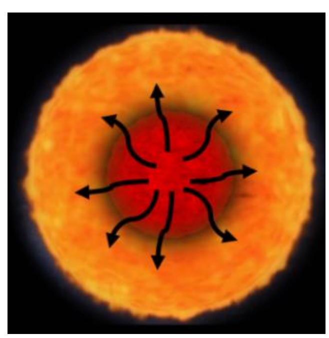
We need to understand what observable effects we can use when a star's fuel runs out.

1. Because the core temperatures are not high enough to fuse helium, once the hydrogen is used up, fusion in the core ceases. Without fusion, there is no nuclear energy source to supply heat to the central region of the star. The long period of hydrostatic equilibrium ends. Gravity again takes over, and the core begins to contract.

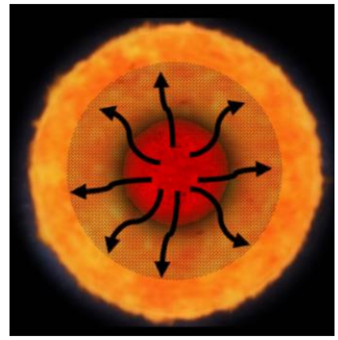




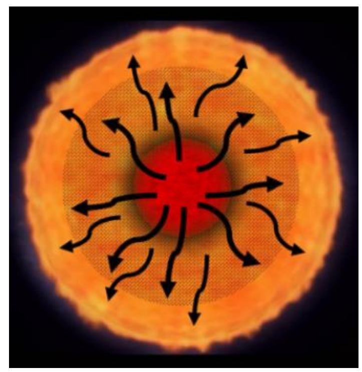
2. As the star's core shrinks, the energy of the inward-falling material is converted to heat. The heat flows outward to cooler regions.



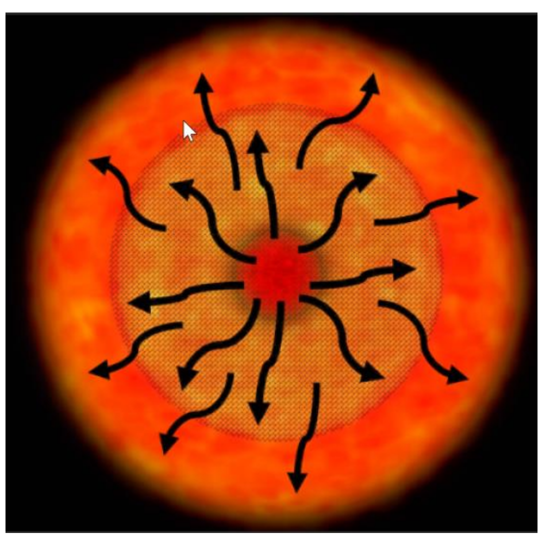
3. The added heat raises the temperature of the layer of hydrogen just outside the core. Once the shell becomes hot enough, hydrogen fusion begins there.



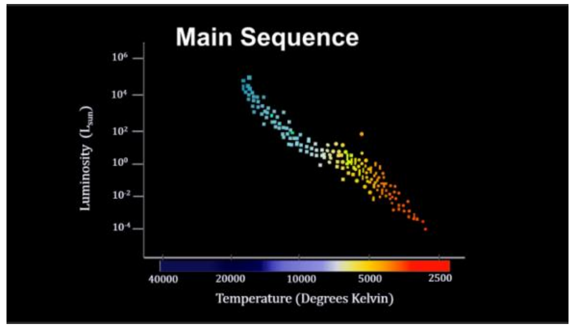
4. Meanwhile, the helium core continues to contract, producing more heat all around it. This leads to more fusion in additional shells of fresh hydrogen outside the core. The additional fusion produces still more energy, which also flows out into the upper layers of the star.



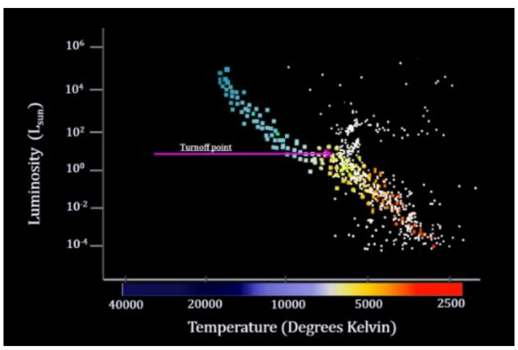
5. Most stars actually generate more energy each second when they are fusing hydrogen in the shell surrounding the helium core than they did when hydrogen fusion was going strong in the core. The first observable result is an increase in the star's luminosity.
6. With all the new energy pouring outward, the outer layers of the star begin to expand. The star eventually grows and grows until it reaches enormous proportions. The expansion of a star's outer layers causes the temperature at the surface to cool. Here we have the second major observable result - the star's surface temperature decreases.



The star becomes simultaneously more luminous and cooler. On the H–R diagram, we see that the star leaves the main-sequence and moves upward (because it's brighter) and to the right (because its cooler).



Detecting this is then a matter of finding stars in a cluster leaving the main sequence. Here's a map of the M67 stars to the H-R Diagram. You can see where the stars are currently moving off the main line. This is called the 'Turn-Off point'. There are no longer any stars in the cluster higher on the main line. The Turn-Off point gives us the luminosity of the stars moving off the main sequence. The mass-luminosity relationship gives us the mass, and with the mass we can calculate the age. For M67 we find that it's around 4 billion years old. We then know the age of all the stars in the cluster.



We estimate

$$L = 10L_\odot$$
$$L / L_\odot = (M/M_\odot)^4$$
$$\tau = 10^{10}(M_\odot / M)^{2.5} \text{ years}$$
$$\tau = 4 \times 10^9 \text{ years}$$

But it does not work for field stars like our Sun. Research into rotation rates and sun spots as age predictors called gyrochronology is on-going. But at this time, we have no way to figure out the age of field stars by examination. If we're going to figure out the age of our Sun, we'll have to examine the material that formed around the sun – the comets, asteroids, moons, and planets including the Earth.